Noesis

The Journal of the Mega Society
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EDITORIAL
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An important ballot is included in this issue. Please tear it out, vote, and send it in.

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THE 10-MARBLES PROBLEM

Chris Cole

Chris Langan argues:

> For example, suppose that the method of filling the box was chosen deliberately to conceal the nature of the prior distribution. E.g., suppose that the prior distribution consisted of 10 white and 10 million nonwhite marbles of various specific colors, but that the 10 white marbles were deliberately sought out and put in the box. Then virtually all continuity between the prior distribution and the subsequent observations has been destroyed, and knowledge of the prior distribution – in which nonwhite marbles were a million times more numerous than the white ones – can only interfere with accuracy. Since we cannot assume that the contents of the box reflect the prior distribution, knowledge of the prior distribution cannot be necessary.

In this paragraph, Chris appears to misunderstand the term "prior distribution." The prior distribution is the distribution of the colors of the marbles produced by whatever selection rule is used to fill the box, not the distribution of the marbles in whatever pool they were selected from. Thus, if only white marbles are selected, the prior distribution is 100% white; if a coin was flipped, you get a binomial distribution, etc.

Chris also argues:

> This brings up a very basic distinction between logic and probability, or deterministic and probabilistic reasoning. Probability does not have to be perfect; it only has to be valid in "most cases." Unlike deterministic constraint, which can be factually invalidated by counterexample, probability is invulnerable to occasional bursts of improbable short-term data. Such deviations are inevitable, and we cannot require probabilistic theorems to forecast every one of them specifically.

Here, Chris appears to misunderstand the term "probability." The theory of probability is derivable from set theory; it is a branch of mathematics; it is no more or less perfect than logic. Statements of probability always are uncertain to some degree, because, like logic, they depend upon the assumptions that are made. This is all that Bayesian Regression has to say; it is really not that big of a deal.

Chris issues this challenge to me:

> I predict that you cannot find **one (1)** professional probability theorist, now working for a college or university in the U.S., who will back your viewpoint ... i.e., who will identify himself fully and say in print that the law of large numbers – or the relationship of frequency to probability that it implies – fails to apply to a closed and finite set of marbles in a box.

And again, Chris appears to misunderstand what the "law of large numbers" means. In a general way, this law states that error decreases as the number of samples increases. Thus, for example, after you select ten white marbles from the box it is more likely that there are only white marbles in the box than it was after you had only selected five white marbles from the box. But the law of large numbers certainly does not say that the odds are precisely .67. As for his challenge, I'll do Chris one better: I'll randomly select a probability theorist and send the problem to him.

But before we waste the effort, maybe this will help. Instead of talking about white and nonwhite marbles, let's talk about boy and girl children. Are you saying, Chris, that if I sample (with replacement) ten children from a family of ten, and all of them are girls, then the odds are .67 that they are all girls? No? How about if I sample them from a classroom? Still no? How about if I sample them from a parking lot? Maybe? How about from the beauty salon? Yes? Why the different answers in different cases? Could it be because you have different estimates of the prior distributions in each case?

HIGH RANGE TESTS

Chris Cole

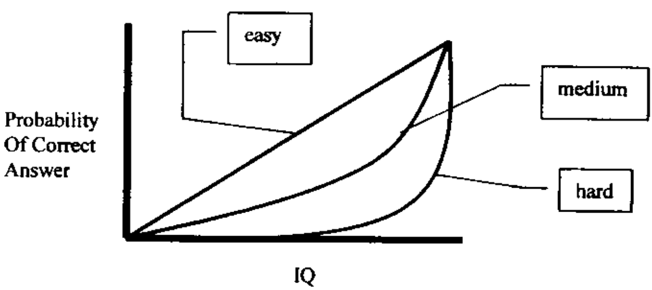
Chris Langan argues:

> You [me] made the following two statements. "It is enough for me to know that the authors of the tests (taken by Paul Maxim) do not claim that they can be used to distinguish at the one-in-a-million level. I think we should believe them." In the interest of fairness, let me add the following equally valid statements. "It is enough for me to know that he authors of the tests (taken by Paul Maxim) do not claim that they cannot be used to distinguish at the one-in-a-million level. I think we should believe them." See? Now things are back in balance. Tests like the Pintner may be "low range" in comparison to tests like the Mega, but their ranges are more than adequate for a sufficiently young (mega-level) child.

In issue 126, I stated that I would not spend time discussing the concept of "range" in testing because I felt the members already understood it. From the above it is clear that at least one member does not. First of all, let me explain why I do not think childhood IQ scores can be used for admission to Mega. A childhood IQ score is frequently computed using "mental age" divided by "physical age," so that a person scoring 200 at the age of ten has done as well on the test as an average person of twenty. However, we also hear that IQ as measured by several popular tests has a mean of 100 and a standard deviation of 16. How can these both be true? The answer is that near the mean (100) the population is roughly normally distributed, with a standard deviation of 16. Out near the Mega level, the distribution looks nothing like the tail of a bell curve, and we certainly cannot conclude that someone scoring 176 on a childhood IQ test is at the one-in-a-million level.

Secondly, the designers of IQ tests are trying to find out where people are near the mean; they are not trying to explore the Mega level. Cynics would point out that this is because there is no market up there. I'm sure that is part of the story, but in addition we should recognize that many of these tests are intended to diagnose learning disabilities, so that, if they deviate from the mean at all, they concentrate on the low side. The purpose of the tests is to distinguish people who are near or below the mean. A test designed to do this must be composed of relatively easy problems. To see why, I have run a simulation. I created three different "tests" – one easy, one medium, one hard. The easy test is composed of 500 easy problems, the medium test is composed of 500 medium problems, and the hard test is composed of 500 hard problems. What is an easy, medium or hard problem? A graph explains it better than words:

Graph of Problem Difficulty



The histograms below show the results of the simulated taking of this test by 10,000 "people." The people were uniformly distributed across intelligence, except two extra points were added at the high end. Intelligence is measured on an arbitrary scale from 0 to 1; a person with intelligence of 0.5 is five times more likely to correctly answer an easy problem than a person with intelligence of 0.1, for example. The asterisks on the histograms represent two standard deviations around the mean for each intelligence level. What the test designer is looking for is to make sure that the lines for 0.5, for example, do not overlap the lines for 0.4 or 0.6.

These histograms show that the easy test does a good job of spreading out the people with intelligence from 0.1 to 0.8, and a poor job above this. The hard test, on the other hand, does a poor job of distinguishing intelligence below 0.5, and better above this.

```
easy:
  0.1      ****
  0.2            ****
  0.3                 ******
  0.4                       ******
  0.5                              ******
  0.6                                    ******
  0.7                                          ******
  0.8                                                 ******
  0.9                                                        ****
  0.95                                                                ***
  0.99                                                                   **

medium:
  0.1 *
  0.2   ***
  0.3     ****
  0.4         *****
  0.5              *****
  0.6                     *****
  0.7                             ******
  0.8                                      ******
  0.9                                                  *****
  0.95                                                             ****
  0.99                                                                   **

hard:
  0.1
  0.2 *
  0.3 ***
  0.4   ***
  0.5       *****
  0.6            *****
  0.7                    *****
  0.8                               ******
  0.9                                            *****
  0.95                                                          *****
  0.99                                                                  **
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Thus, we do not need to be explicitly told by the designers of the Pintner test, or any other standard intelligence test, that they are not valid in the Mega range. If they were valid in the Mega range, then they would be useless in the normal (100) range. It is simply impossible to design a test that is valid in both ranges. This has nothing to do with the number of people that took the Pintner test, how big the norming sample was, what the intended age of the testees was, etc. To claim otherwise is bad science.

PLACE
POSTAGE
STAMP HERE

Jeff Ward
13155 Wimberry square #284
San Diego, CA 92128

MEGA SOCIETY BALLOT

Enter your name: ______________________________________

Indicate your vote on any or all of the following proposals and mail your ballot by September 15 to:

Jeff Ward
13155 Wimberly Square #284
San Diego, CA 92128

Vote for ONLY ONE of the next three proposals. Indicate your choice in the box to the left.

Enter 1, 2, or 3: [ ]

1. The Bylaws of the Mega Society shall be as published in *Noesis* issue 123.

2. The Bylaws of the Mega Society shall be as published in *Noesis* issue 123, amended as proposed by Kevin Langdon in *Noesis* issue 125.

3. The Bylaws of the Mega Society shall be:
The Mega Society shall have three positions elected by a majority vote of those members casting valid ballots: Administrator, Editor, and Publisher. The term of these positions shall be two years. The Administrator shall handle administrative matters such as elections and applications for membership. To be admitted to membership, a person must have scored at or above the one-in-a-million level on a test of general intelligence, and must pay an initiation fee of $15. The Publisher shall publish and the Editor shall edit the newsletter. Subscription fees for the newsletter shall be set to an amount sufficient to cover the cost of publication and distribution. Bylaw changes and major decisions regarding the governance of the Society shall be decided by a majority vote of those members castings valid ballots.

Vote YES or NO for the following proposals.

YES	NO	Jeff Ward shall be the Administrator of the Mega Society.
YES	NO	Chris Cole shall be the Publisher of the Mega Society.
YES	NO	I nominate myself to be the Editor of the Mega Society.
YES	NO	To be acceptable as an admission test for the Mega Society, a test must be credibly claimed by its author(s) to be able to distinguish intelligence at the one-in-a-million level.
YES	NO	The following tests are to be used for admission to the Mega Society: The Mega Test by Ron Hoeflin The Titan Test by Ron Hoeflin

Tear out, staple, and mail TODAY!

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Dear Rick Rosner,

I wrote to Dr. Hoeflin about a month ago and told him I had a formula for the maximum # of volumes generated by n inter-penetrating cubes for n=3,4,... . Now, after two failed attempts, the first of which I sent to Hoeflin, I now have derived a formula for this for n=1,2,3,4... . This formula has, from a mathematical point of view, a beautiful and elegant property which was quite serendipitous, if one accepts, as I believe one must, that the max. # for n=2 is 23. In lieu of a strict proof, although something approaching one, I believe this should clinch the analysis. Hoeflin said he would not mind if members of the Mega Society learned of this formula, that a few members had been working on the problem, and gave me your name and address as Editor, so I decided to send it along. I cannot guarantee that there is such a formula for all n, although it works for up to n=6, by my calculations, but if there is such a formula, this must be it, for it has all the right properties. To save time, for the next two paragraphs, I quote from a letter sent to R. Fred Vaughan, Editor of "Gift of Fire", journal of the Prometheus Society.

'Well there it is, and it looks so simple! What can be learned from this problem? The first difficulty that may have caused many to balk at it was simply not knowing where to begin. But the real difficulty lies, not in the number of steps, but in the number of assumptions that need to be made, without feedback. This touches upon something profound, Meiner Meinung nach, of knowing when to have faith in one's intuition, and then having strong confidence in it, when there are no or few confirmations to guide one. This is likely to be a part of all deep problem-solving. And the final argument must be partly heuristic, rather than a complete proof, placing an even greater demand for faith or confidence in one's intuition, and a type of argument that even professional mathematicians seldom adopt. (However, there is rigor enough, I believe.)

What are the basic assumptions that one must make for this problem, in order to be delivered from blind alleys? They are simple, but essential. To me, the first and most basic would be that the first cube is the base cube and does NOT move. (Nothing but confusion results from trying to rotate all three, or all n, cubes.) Then the volumes are all generated exterior to the base cube, with almost all of the volumes generated on 4 faces of the base cube, rather than all 6, for no one rotation can accommodate all 6 faces. (One must early on convince oneself that nothing close to a maximum # can be generated from the interior of the base cube.) Then one turns the other cubes so that they appear as diamonds, viewed head-on, and uses the resulting wedges to generate the great majority of the volumes, one part generated by the line of the wedge on the 4 faces, the other volumes of this rotation generated by 4 planes/cube cutting through 8 corners of the base cube and jutting out, slicing through these volumes for the next cube and jutting out again, etc. Then one must recognize that there are three separate processes at work in these rotations, which can, rather easily individually be maximized, but the integrity or harmony of the overall process is not violated by taking them separately and adding them together. Finally, one should recognize that nowhere was it stated that the cubes must have the same volume, so that this is not a valid constraint.

As background for my derivation, I cite, "Induction and Analogy in Mathematics", Volume I of "Mathematics of Plausible Reasoning", Chapter III, "Induction in Solid Geometry", especially that about the table in about the middle of the chapter and of the middle column. From that, I make use of what I take as a lemma, that where the points of contact of a geometric entity go up by one, that geometric entity also increases each time by one unit. For those volumes generated on the faces of the base cube, I make use of a topological argument, that if one has n lines, none of which are parallel, each line, in a large enough space, will divide every other n-1 times, for nxn total divisions. Thus, this process can be scaled down to fit a closed area and give the same result, by suitably changing the angles, and the points of intersection increase as $\binom{n-1}{2}$ + # DIVISIONS OF LINE $(n-1)^2$

BY G. POLYA

And one must not forget the inner, remaining volume of the base cube, i.e. to add 1.

The beautiful and serendipitous result of this formula is that, if one takes the difference between the # generated by 2 cubes and 1 cubes to be 22, then the formula says to take this number and multiply it by the # of interactions of the generating cube taken 2 at a time, plus 1, to get the maximal # of volumes for n inter-penetrating or interacting cubes, i.e.,

TOTAL # MAX. VOLUMES ~~OF~~ FOR N CUBES $= 22\binom{N-1}{2}$ ~~+ 1~~. TO BE ADDED TO # N=2

FORMULA IS $\underline{11N^2 - 11N + 1}$ TOTAL # OF MAX VOL GEN BY N CUBES $= 22\binom{N}{2} + 1$

∴ FOR VOLUMES GENERATED ON FACES B.C. $(N-1)^2 + \frac{(N-2)(N-1)}{2} =$

$3N^2 - 7N + 4/2$ MULTIPLY × 4 →

$\underline{6N^2 - 14N + 8}$

FOR THE 8 CORNERS, INCREASES AS 2, 5, 9, 14, 20, ...

$8\left[\binom{n}{2} - 1\right] = 4N^2 + 4N - 8$

VOLUMES GENERATED BY JUTTING PLANES ON THE 2 OTHER BASE FACES

INCREASES AS 1, 3, 6, 10, 15, 21,

∴ $\underline{2\binom{N}{2} + 1 = N^2 - N + 1}$

$6N^2 - 14N + 8$
$4N^2 + 4N - 8$
$N^2 - N + 1$

$\boxed{11N^2 - 11N + 1}$

Dear Rick Rosner,

A few final thoughts. I want it to be clear that I have not given a proof, but only a conjecture. And because one can never prove a negative, a conjecture is all one can ever hope for. Dr. Hoeflin wrote that two members of the Mega Society were seeking a proof for n=3. Perhaps his choice of word was not as precise as it should have been; "demonstration" or even "rigorous demonstration" would have placed such activity within the domain of the possible, but not "proof". Thus I offer no excuse for not providing a proof for my formula, and insist that it not be seen as a flawed attempt to move in that direction. But it is a conjecture, and I believe, if I say so myself, a damn good one.

The criteria of a conjecture in mathematics is that it be mathematically reasonable. This, it seems to me, can be achieved in essentially two ways. The first is to enumerate a very large number of instances of the conjecture without one failure. This is the method of Goldbach's Conjecture, and from a slightly different perspective, the hypothesis that the number of twin primes is infinite. This method is denied, even if one had a computer that could be programmed to count these volumes for very large n interacting cubes, as you have not proven that there is no way in which higher numbers of volumes could be generated---and this problem, as a proof, is always open-ended, as to possibilities. The other method or tack is to set up some reasonable assumptions for such maxima, and establish a proof, based upon those assumptions. So I now sketch a proof, that was not fully developed before, and which can be 'filled in' in those places where needed, by the members of your Society, for my formula.

The key is in the increase in the rate of increase of the generated volumes/plane or faces. For the first rotation of the wedges on 4 base faces, there are 3 processes at work. The first is by their edges, acting like lines, and n 'lines' each being divided into n pieces by the other n-1 'lines'. This, of course, is formulated as $(n-1)^2$ But the increase in the rate of increase of n^2 is 2. The other 2 processes, the # of intersections of the (n-1) edges and the 2 slashing of corners and jutting out, followed by intersecting maximally and jutting out again etc., both have an increase in the rate of increase of 1 for each of the 3 remaining processes (two of which are symmetrical). This result is established, following Polya, (reference previously given) that where the points of contact increase each time by 1, the increase in the rate of increase of the geometric entities generated is 1. Thus one has, for one plane in the first rotation 5 times the increase in the rate of increase by 1, for all 4 planes, the gives 20 times this arithmetic entity, the other 2 planes have a similar increase in the rate of increase. This gives 22 times this increase in the rate of increase by 1 each time +1 for the unaffected part of the base cube. But there is one way that such conditions can be expressed mathematically, and that is $22\binom{N}{2}+1$, , which is the formula I have offered.

$= 11N^2 - 11N + 1$

Cordially,

Ronald Penner

Ronald Penner

APPENDA

\# OF EDGES INCREASES AS $(N-1)^2$

\# OF INTERSECTIONS " $\binom{N-1}{2}$

THE CORNER SLASHERS + JUTTERS " $\binom{N+1}{2} - 1$

WHY THE $\binom{N+1}{2}$? BECAUSE FROM THE FIRST INTERACTION A POINT OF CONTACT OR INTERACTION IS ESTABLISHED, IT DOES NOT LAG BY ONE INTERACTION LIKE THE REMAINING PLACES DO AND MUTATIS MUTANDIS FOR N-1.

$$\therefore \quad (N-1)^2 + \binom{N-1}{2} + 2\left[\binom{N+1}{2} - 1\right] = 2N^2 - 4N + 2 + N^2 - 3N + 2 + 2N^2 + 2N - 4/2$$

$$= 5N^2 - 5N/2 = 5\binom{N}{2}$$