



Noesis

The Journal of the Mega Society

Issue #217, July 2026

About the Mega Society

The Mega Society was founded by Dr. Ronald K. Hoeflin in 1982. The 606 Society (6 in 10^6), founded by Christopher Harding, was incorporated into the new society and those with IQ scores on the Langdon Adult Intelligence Test (LAIT) of 173 or higher were also invited to join. (The LAIT qualifying score was subsequently raised to 175; official scoring of the LAIT terminated at the end of 1993, after the test was compromised.) A number of different tests were accepted by 606 and during the first few years of the Mega Society's existence. Later, the LAIT and Dr. Hoeflin's Mega Test became the sole official entrance tests, by majority vote of the membership. Then, Dr. Hoeflin's Titan Test was added. (The Mega Test and Titan Test were also compromised, so Mega Test scores after 1994 and Titan Test scores after August 31st, 2020 are currently not accepted; the Mega and Titan cutoff is 43 - but either the LAIT cutoff or the cutoff on Dr. Ronald K. Hoeflin's tests will need to be changed, as they are not equivalent.) By majority vote, The Mega Society now does not accept previously-qualifying scores on The Ultra Test after May 27th, 2026. The Mega Society still accepts qualifying scores on The Hoeflin Power Test, however. The Mega Society publishes this irregularly-timed journal, *Noesis*.

Answer sheets for various tests can be emailed to ultrapowertest@gmail.com; the scoring fee for each test is \$10 USD, payable via Stripe.

<https://megasociety.org/#admission>

The society also has a (low-traffic) members-only email list. Mega members, please contact one of the Mega Society officers to be added to the list.

For more background on Mega, please refer to Darryl Miyaguchi's "A Short (and Bloody) History of the High-IQ Societies" —

<http://miyaguchi.4sigma.org/BloodyHistory/history.html>

—and the official (designed) Mega Society page,

<http://www.megasociety.org/>

Noesis is the journal of the Mega Society, an organization whose members are selected by means of high-range intelligence tests.

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Noesis #217, July 2026

Editorial

Ken Shea

While the current issue starts with well-trodden tours of Roman and Greek antiquity, the ground quickly becomes more adventurous with excursions leading to, for example, black holes, quantum field theory, experimental mathematics, cryptanalysis, and artificial intelligence.

Before embarking on the journey, however, sad news a bit closer to home will have to be imparted: Shortly after the publication of the last issue, the Mega Society learned about the sudden passing of long-time *Noesis* editor Kevin Langdon at the age of 82.

Kevin Langdon's Wikitia page lists Kevin's date of birth as May 3rd, 1943 and indicates Kevin was a polymath, writer, musician, and psychometrician. The initial report of Kevin's passing - the details were pieced together at a later time - stems from September of 2025.

Kevin pioneered in the field of high-I.Q. testing and proved instrumental in shaping numerous high-I.Q. societies still active today. Kevin Langdon and Ronald K. Hoeflin co-founded the Triple Nine Society - with Richard Canty, Ronald Penner, and Edgar Van Vleck - in 1978, for instance.

A year later, the eponymous Langdon Adult Intelligence Test (L.A.I.T.) was published in the April 1979 issue of *Omni* magazine. Ronald K. Hoeflin's legendary Mega Test (April 1985) and fearsome Titan Test (April 1990) would subsequently grace the magazine's pages.

https://megasociety.org/LAIT_April_1979_Omni.pdf

Kevin Langdon first assumed the role of *Noesis* editor in the November 1997 issue of the journal, *Noesis* #135.

<https://megasociety.org/noesis/135.pdf>

The 'editorial standards' Kevin Langdon established in the opening editorial seem basically sound after all these years:

'Editorial standards: no unproven accusations, no ad hominem arguments, no dot-matrix print, reasonably good taste, reasonably interesting content, opinions supported by evidence. The Editor will sometimes request that one or more points in a submission be clarified or expanded. Please don't be offended by this; it means I think your work is good. Try to anticipate anything that may be unclear to readers of *Noesis* and make repairs prior to publication. If the repairs are to content rather than English mechanics, you'll have the opportunity to make the changes yourself.'

In my view, two sentences preceding Kevin's 'editorial standards' actually typify Kevin's overriding *ethos* more crisply - 'These are my views. I welcome your views.'

In the current issue's first contribution, Chris Cole has, yet again, joined forces with Anthropic's Claude to historically deconstruct a unit of measurement in "Why are there 5,280 feet in a mile?"

Next, award-winning playwright and performer Alex Lyras, the creative force (writer and director) behind the recent play *Aristotle/Alexander*, reflects on the process of bringing ancient philosophy to the modern stage in a thought-provoking interview with Scott Douglas Jacobsen.

The Nigerian human rights activist and lawyer Dr. Leo Igwe, then, chronicles hurdles in the fight to advance secular values in Nigeria in an interview with Scott Douglas Jacobsen titled, "Making Witch Hunting History: Dr. Leo Igwe's Fight for Justice."

After that, Dr. İzzet Sakallı returns for another interview with Scott Douglas Jacobsen, who summarizes the exchange thus: 'Dr. İzzet Sakallı addresses observational and theoretical strategies for testing beyond-general relativity physics using black-hole imaging, lensing, and gravitational-wave data.' Mario Carlos Rocca, an Italian experimental physicist, also enjoys an interview with Scott Douglas Jacobsen, immediately afterwards. 'The interview concerns mathematical structures used in quantum field theory that generalize Schwartz distributions, focusing on tempered ultradistributions and ultradistributions of exponential type.' Easy, right?

Moving on, a few Mega Society members, including Chris Cole and Werner Couwenbergh, worked alongside Anthropic's Claude to answer problems, with a few remaining open by the provisional end of the project, about interpenetrating cubes. These two contributions are straightforwardly titled "How Many Regions Do Overlapping Cubes Make?" and "Six Glass Cubes: A Week of Experimental Mathematics with a Team of AIs."

(Please see 'The Collaboration, Honestly Described' starting on page 87 for more details.)

Chris Cole, then, obliges cries for an encore by delivering "The Feynman Ciphers: A Cryptanalysis Story, 1945-2026." Cf. <https://en.wikipedia.org/wiki/Cryptanalysis>

Afterwards, Rick Rosner sits down for two wide-ranging interviews with Scott Douglas Jacobsen. Topics include: religion, creationism, AGI, LSD, social class, art, and war.

Finally, Omega Society member and former *Noesis* contributor Adam Kisby leaves readers with a remembrance of dear friend Richard May, who somehow still manages to have the last word.

The next issue of *Noesis* will be published in March of 2027.

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Why are there 5,280 feet in a mile?

Chris Cole and Anthropic's Claude Fable 5

Ask an LLM and you get the standard story:

The reason there are 5,280 feet in a mile goes back to ancient Roman times and evolved through English history. The Romans used a unit called the "mille passus," or "thousand paces," which was based on a pace of about 5 feet (two steps, roughly). This worked out to around 5,000 Roman feet. When the English adopted this concept, they adjusted it to fit their own system of measurements.

In medieval England, land was often measured using the furlong, which was based on the length of a plow furrow—about 660 feet in the English system. There was also an older Saxon unit, the "mile," which varied but was often around 5,000 feet. Over time, lawmakers wanted to standardize these units. In 1593, during Queen Elizabeth I's reign, a statute redefined the mile as exactly 8 furlongs to tie it neatly to the agricultural measurements already in use. Since a furlong was 660 feet, 8 times 660 equals 5,280 feet.

So, the 5,280 feet in a mile is a blend of Roman influence and English practicality, refined to align with the furlong. It's quirky, but history tends to leave us with these oddities!

So the mile is 5,280 feet because it is eight furlongs, and a furlong is 660 feet. But this merely pushes the question back a step: why is a furlong 660 feet? The traditional answer is that a furlong is the length of an average furrow plowed by an average team of oxen in an average field. Notice what this "explanation" accomplishes: it is precise enough to sound like an answer and vague enough to prevent any further inquiry (Quine would have admired the maneuver). No one can produce the average field or the average ox, so no one can check.

Take the foot as a given unit and look at the actual structure of the old English measures:

rod (also pole or perch) = $5 \frac{1}{2}$ yards = $16 \frac{1}{2}$ feet
chain = 4 rods = 22 yards = 66 feet
furlong = 10 chains = 660 feet
mile = 8 furlongs = 5,280 feet

A factor of 11 runs through the entire system: $5 \frac{1}{2}$, 22, 66, 660. Oxen do not plow in multiples of 11. Where does it come from?

Start with a different question: who actually cares, to high precision, how long a mile is? Not the plowman; his furrow ends where the oxen tire. Not the traveler; a journey of twenty miles is not

spoiled by a mile that runs one percent long. The people who care are the people who run races. A record means nothing unless the distance is the same from one track to the next, so a race course must be standardized to better than one percent.

Suppose, then, that you want to build a track of a fixed, reproducible length—shorter than a mile, so races of various distances can be run in laps. The natural shape is a circle. But a circle presents a problem: you cannot measure its circumference directly with any accuracy. What you can measure, with nothing but a stake and a rope, is its diameter. So you need the circumference to come out to a round number when the diameter does—that is, you need a good rational approximation to pi.

The continued fraction of pi is $[3; 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, \dots]$. Its first convergent, $22/7$, is the best rational approximation to pi with a small denominator, and it is a remarkably good one: it is within 0.04% of pi—twenty-five times better than our one-percent requirement. (The next convergent, $333/106$, buys extra precision that no rope-and-stake surveyor could use.)

Now the 11 explains itself. If the circumference is to be $22/7$ times the diameter, then for both to be whole numbers the diameter must be a multiple of 7 feet and the circumference a multiple of 22 feet. A circle 210 feet across has a circumference of $210 \times \pi = 659.73\dots$ feet—one furlong, to within about three inches. Drive a stake, swing a 105-foot rope, mark the circle, and you have laid out a standardized furlong track anywhere in England, accurate to 0.04%. Only lengths divisible by 11 admit this construction, because $22/7$ is such a good approximation to pi.

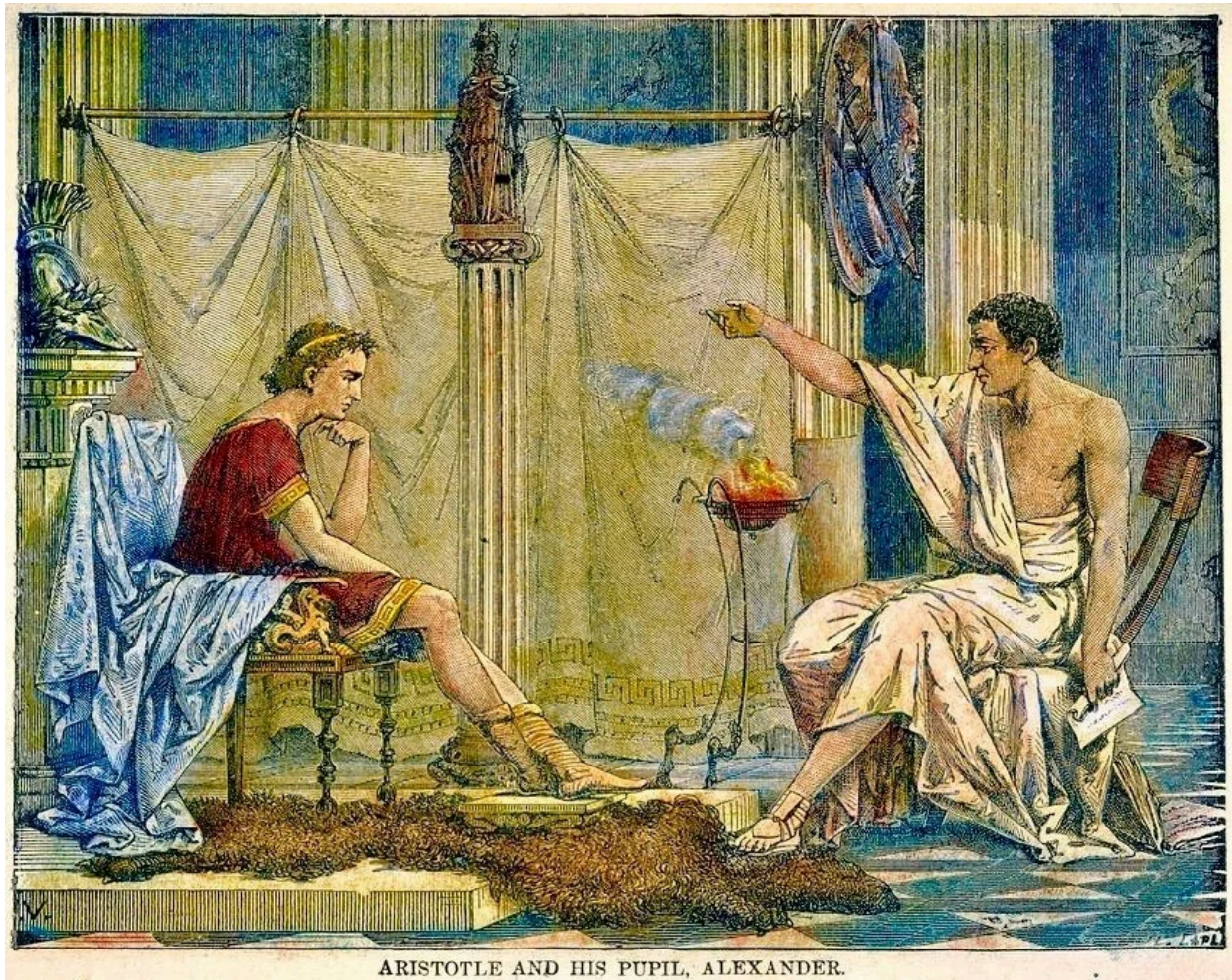
The smaller units wear the same fingerprint. A circle 7 yards in diameter has a circumference of one chain—22 yards—to within an inch. The chain is not just divisible by 11; it is $22/7$ made flesh.

Hence 660 feet in a furlong. Hence 5,280 feet in a mile.

Our forebears were smarter than history reflects.

Aristotle/Alexander: Philosophy and the Fragility of Democracy

Alex Lyras & Scott Douglas Jacobsen



ARISTOTLE AND HIS PUPIL, ALEXANDER.

Award-winning playwright and performer Alex Lyras brings ancient philosophy to the modern stage in *Aristotle/Alexander*, a compelling production presented by the Center for Inquiry in Los Angeles. The play dramatizes the formative relationship between Aristotle and a young Alexander the Great, exploring mentorship, ethics, and political ambition themes. Through deep philosophical lineage—from Socrates to Plato to Aristotle—Lyras contrasts ideals with pragmatism and reason with conquest. Rooted in historical depth yet charged with modern relevance, *Aristotle/Alexander* invites audiences to reflect on the fragility of democracy, the pursuit of legacy, and the moral challenges of leadership, both then and now.

Scott Douglas Jacobsen: Today, we are discussing the work of Alex Lyras—an award-winning playwright, director, and performer known for creating intellectually rich, emotionally resonant solo performances that explore themes of identity, philosophy, and sociopolitical complexity.

His most recent production, *Aristotle/Alexander*, presented in collaboration with the Center for Inquiry in Los Angeles, dramatizes a speculative and historically inspired encounter between the philosopher Aristotle and his young pupil, Alexander the Great. The play examines enduring political and ethical dilemmas framed through the lens of mentorship and the shaping of a future world conqueror.

Lyras's work is acclaimed for its combination of depth, humour, and sharp commentary on the fragility of democracy and the abuse of power. A longtime presence in the Los Angeles and New York theatre scenes, Lyras brings a unique voice to contemporary playwriting, skillfully blending classical ideas with urgent modern questions.

There are many stories about Aristotle, Plato, Socrates, and the lineage of philosophical teaching that come down to us from antiquity. What inspired you to dramatize the relationship between Aristotle and Alexander the Great?

Alex Lyras: I was surprised this story hadn't been dramatized in greater depth. We've all heard about how Aristotle tutored Alexander, but it's rarely explored dramatically or philosophically sustainably. Historical fiction often mentions it in passing, but writers may avoid digging into the philosophy because it's dense and complex.

I studied philosophy at Bucknell University and read it throughout my life. I found myself returning to this unique setup again and again: the world's most brilliant philosopher paired with the world's most ambitious military mind. It's a remarkable relationship—one rooted in education and power—and yet no one seemed to be exploring it deeply on stage.

I've always been interested in developmental narratives, especially early formation stories. When I started researching, I didn't find much dramatizing their relationship with nuance or complexity. I realized this was not only philosophically fertile ground but also a compelling theatrical concept. I imagined *Aristotle/Alexander* as a playbill title, something you might see at Lincoln Center or a major repertory theatre. It felt like the kind of title that would grab people: two names, one relationship, and centuries of tension.

So, it was a quick sell. What happens when the greatest mind in philosophy is charged with educating the most significant figure in military history? That tension—between intellect and ambition, reason and conquest—was irresistible.

Jacobsen: What stood out to you in the narratives about Aristotle? What about Alexander the Great? Did you compare their ideas or trajectories with those of Plato or Socrates to understand how those philosophical lineages evolved into the narrative you've created?

Lyras: The play tackles these questions gradually. How Plato created the character of Socrates in his dialogues. How Aristotle broke from Platonic Idealism and leaned more into empirical science, and how it all culminated, for better or worse, in Alexander's development during his all too brief reign. The philosophical throughline contrasts ideals and pragmatism, ethics and empire, reason and domination. All of that informs the dramatic tension in the play.

[Editor's Note: Raphael's *The School of Athens* depicts Plato (left) fingering heavenward and Aristotle (right) palming downward - the latter insisting on earthly (nay, sublunary) matters?



Jacobsen: What stood out to you about Aristotle?

Lyras: Aristotle was originally from the north of Greece. Ancient Stagiera is not far from ancient Macedonia, which is basically where Thessaloniki is today. So, he was a northerner who moved to Athens, and was a bit of an outsider. But he was formidable. They called him “the brain” when he was studying at Plato’s Academy— everyone recognized his brilliance. He quickly rose to prominence and became Plato’s star student.

But after Plato died, politics took over, and Aristotle was passed over to lead the Academy. Instead, the Academy was handed to Plato's nephew, Speusippus. Nepotism is also an ancient tradition. After Aristotle was snubbed, he left Athens and began doing independent research on the island of Lesbos, which he likely wanted to do anyway. What struck me is that someone so universally recognized as a genius did not receive what many would consider his just reward. That moment in itself is so dramatically compelling. You're not meeting him at his height—you're meeting him in a moment of rejection and transition.

He returns to Macedonia, which is... nothing like Athens—let's just put it that way. While Athens was in decline politically, it remained Greece's cultural and intellectual heart. "The school of Greece." Even during the Roman period, people still went to Athens to study philosophy, sculpture, and architecture. But Aristotle chose to leave.

And then there's Alexander—just 13 years old when they crossed paths. Their mentorship lasted until Alexander was about 16. You're not going to get a juicier moment for a character study: a teenager who is, in some ways, significantly entitled but also intelligent and cultured, growing up as a prince in a court that received scholars, emissaries, and artists. That dynamic—the philosopher in exile and the royal prodigy—creates a powerful dramatic conflict.

Jacobsen: How did you think about Socrates and Plato in the context of this? Did you find meaningful philosophical contrasts?

Lyras: Definitely. You have to contextualize it and recognize that Aristotle is a very different kind of thinker. Plato is theoretical—he's concerned with ideal forms and metaphysical structures. He's talking about "the Good," the cave metaphor, and "Divided Line" and other abstract ideals. These concepts are foundational for ethics.

But when you get to Aristotle, suddenly, we're in the realm of practical ethics. He's empirical as opposed to conceptual. He collects fifty constitutions from various city-states and begins comparing them to understand how laws are written and how societies function. He starts categorizing, organizing, and trying to make sense of the world through observation. He invents taxonomy to keep his extensive biological research organized. Then, he develops logic—the foundations of reasoning, in order to explain it all rationally to others. He didn't want to speculate about ideas in general; he aspired to analyze them systematically. *Analysis* means "unravelling" in ancient Greek. I love that....

Jacobsen: Was that the origin of science as we think of it today?

Lyras: In many ways, yes. Aristotle laid the groundwork for empirical reasoning. People were still steeped in mythology at that time—offering sacrifices and libations to the gods. Plato was engaged in esoteric theory. Aristotle, by contrast, grounded philosophy in causality. He introduced a method of questioning a vast number of subjects which becomes the foundation for scientific enquiry for the next thousand years.

Jacobsen: Aristotle wanted evidence and proof for what constitutes good ethical behaviour, correct?

Lyras: In a way, yes. You're dealing with someone who's no longer just standing on the *Pnyx*—the hill in Athens where citizens gave public speeches—and trying to persuade people rhetorically. That wasn't his game. He wasn't interested in persuasion for its own sake. He aimed to gather evidence, establish causal relationships, and leave behind a systematic body of work for others to study—especially those who hadn't studied directly at his school.

He was also a prolific writer. Most of what we have today from Aristotle are compiled lecture notes or student transcriptions, but they've endured because he took the time to be methodical in his thinking. That's a legacy of structure.

Jacobsen: Teacher-student dynamics can vary widely. How do you think Aristotle saw Alexander? How did Alexander see Aristotle? And what does that tell us about their leadership styles and eventual legacies?

Lyras: That's a loaded question because we don't know for sure. But there's plenty of material out there to speculate with some confidence.

Right before Aristotle received the invitation from King Philip II to tutor his son, Alexander, he was living on the island of Lesbos, doing detailed marine biological research. Athens had become so factionalized and critical of competing schools of philosophy that he essentially retreated from it. On Lesbos, he created a kind of proto-laboratory where he could work in peace. He was in a stage of inductive reasoning—collecting data and making observations to draw hypotheses.

That contrast is essential. In logic, we often talk about deductive reasoning, where we move from a general premise to a conclusion. But Aristotle, particularly in his biological work, practiced inductive reasoning, moving from observation to hypothesis. In the play, I describe this period as one of youthful optimism. He was probably about 40—middle-aged by ancient standards—but he was entering a fertile phase of intellectual maturity.

He was also left alone long enough to do the work without constant criticism, which must have been life-affirming. Then he goes to Macedonia, which, intellectually, was years behind Athens in terms of intellectual sophistication.

Jacobsen: So, how does this relate to leadership?

Lyras: It's central. Aristotle was stepping into a new phase of leadership himself—becoming not just a transmitter of inherited philosophy but an originator of his own system. What he was doing was totally foreign to most people. They didn't understand it. Many thought his thinking was scattered—jumping from astronomy to marine biology to ethics and politics. They wrongly believed that you had to be an expert in one thing. But one of my favorite quotes from Aristotle,

to paraphrase, is “Do we call the bee scattered, for landing on all flowers and sipping the best from each?”

Aristotle’s polymathic nature was a strength, not a weakness. His leadership, pedagogically speaking, was about breadth and synthesis. On the other hand, Alexander was raised in a world of hierarchy, military discipline, and rather brutal conquest. So what happens when a synthetic, reflective thinker teaches a brilliant, ambitious prince? That tension is core to the drama.

Jacobsen: So this is the beginning of Aristotle’s creation of what could be considered the first university—an institution where one could study a wide range of subjects.

Lyras: Yes, exactly. It’s the early model for comprehensive education—what we’d now call a university. Aristotle began building this foundation of interdisciplinary study. The Greek word for university is “πανεπιστήμιο” or pan-episteme, which literally translates to “across knowledge.”

As for Alexander, I push the envelope a bit in the play, portraying him as highly entitled and wildly arrogant. But in all likelihood, he was more sophisticated than that. Clearly he was exposed to far more than the average adolescent up north. His father, King Philip II, had already begun consolidating and expanding Macedon into an empire, and he was deeply invested in endowing Alexander with the intellectual patina he never had.

Philip invited leading thinkers and artists to the Macedonian court. Euripides had been brought north to write for the court under earlier kings. Herodotus had also travelled there. Other prominent Athenians were welcomed and treated like royalty. So Alexander grew up surrounded by culture and intellectual energy from Athens and beyond.

Militarily, he also witnessed Philip’s flexible strategies—sometimes diplomatic, sometimes forceful. So, Alexander came of age with various influences that shaped him as a thinker and a leader. He had a vision, as we say.

Jacobsen: That reminds me of exercises we did in drama class—like taking a traditional Shakespearean play and setting it in the 1950s or another specific decade. The context would change, but the core dynamics remained powerful. You’re doing something similar: starting with an ancient historical premise and bringing out its modern relevance. How do you ensure—although it almost seems guaranteed—that narratives like this remain relevant today, especially the lesser-known ones?

Lyras: That’s a great question. All I can say is that the more I study the Classics—especially this period around the rise of Hellenism—the more I see how timeless it is. Even earlier, we find incredibly resonant ideas during the Golden Age of Greece.

I think about the Mark Twain quote: “History doesn’t repeat itself, but it often rhymes.” And that’s how this feels. What are the political questions we’re grappling with today? They were wrestling with those same ones 2,500 years ago.

How do you govern a polis—a city-state? Or today, a nation? Are there different rules for people in power? Do those working for the powerful receive special advantages? Is it better for the “average” citizen to be disengaged from democracy simply because they’re overworked or underinformed? Someone juggling two or three jobs doesn’t have the time or energy to dive into a candidate’s platform or philosophy. They are too easily manipulated. This is why Socrates hated democracy. He did not have much faith in the average person to be discerning enough to make long term decisions.

These questions haven’t gone away. They’re cyclical. Plato talks about this in *The Republic*—around Book VIII. He outlines how democracies begin: pure, idealistic, and focused on the collective good. But over generations, the people who inherit power do not earn it through struggle or civic commitment. They begin to see the personal benefits of leadership, and bureaucracy grows.

Then, the next generation, in turn, becomes even more disconnected from civic responsibility—and power becomes the end goal. That pattern is not limited to ancient Greece. It’s everywhere, and it keeps repeating in new forms.

We want power. We want favours. And eventually, that devolves into tyranny—one person saying, “I’m getting rid of everyone else, and I’ll do it myself.” This pattern repeats. It’s not unique to Greece.

You can trace it through various ancient tyrannies—some benevolent, some brutal—from Egyptian and Mesopotamian rulers onward. The cycle is always the same: it hinges on the political philosophy of the person in power. We’ve seen it recur in modern history as well—in our own country, during the French Revolution and the Russian Revolution. These are cycles of rise, decay, collapse, and reinvention.

The more committed I was to being historically accurate—doing justice to the specifics of antiquity—the more the work began to resonate with today’s political climate. The more precise I was about the issues back then, the more people responded, “You must be talking about today.”

You can read letters from individuals who lived then and be shocked at how contemporary they sound. That’s the power of historical drama—it rhymes with the present.

Jacobsen: Let’s add one or two more questions here. Why do you personally think democracies are fragile? And what did some of the ancients, like Aristotle, say about this? Is their view different from modern thinkers?

Lyras: Sure—great question. Democracies are fragile because they are, at heart, social contracts. Thinkers like Hobbes, Hume, and John Stuart Mill wrote about this centuries later, but they were building on foundations laid by Plato and Aristotle.

A democracy only works when there is a mutual agreement to act in good faith. It requires cooperation and shared values. If someone enters a democracy to gain power for personal

benefit, they can start to erode the system from within. That self-serving motive becomes contagious. Soon, everyone in power is focused on personal gain, which becomes the new social value.

That is when democracy starts to break down. Post–World War II, and even through the 1950s and 60s in the U.S., there was a greater collective awareness—a sense that we were trying to do what was best for the nation as a whole, whatever that meant in practice. But greed has always been with us. It seems to be a persistent element of human nature that resists elimination.

When enough people in power begin taking advantage of the system, others follow suit. The public begins to lose faith. That's terrifying because democracy is not a given—it's an idea, and we have to believe in that idea for it to function. Without that shared belief and a willingness to cooperate, the whole thing becomes shaky.

Jacobsen: What are some of your favourite quotes from Aristotle or Alexander the Great?

Lyras: Aristotle collected many insights, but one that resonates is: “At his best, man is the noblest of all animals; separated from law and justice, he is the worst.” That encapsulates Aristotle's view of honour and nobility and the dangers of unrestrained power. His point was that we must live in a polis, a civic community governed by laws and justice. And those laws apply to everyone.

He also famously said, “Man is a political animal.” Of course, today, we would say *humankind*—people are political beings. We are meant to live together in communities. Human beings are not meant to be isolated in the mountains. We thrive in collectives because rationality and logic allow us to do far more together than we could alone.

Here's another: “Educating the mind without educating the heart is no education at all.” This is particularly important in the political context. Aristotle believed that it's not enough to know how to win wars or manipulate your enemies—you need to know how to govern and how to build a city after the conflict ends.

This was a central message he tried to impart to Alexander. War is not the end goal; ruling justly and building a functioning society are. That quote is essential to the themes of the play. Another one I love is, “Excellence is never an accident.” He emphasized that virtue must be trained—just like the sword. In the play, I draw this parallel directly: You train with your sword, but you must also train ethically. Making the right choices and acting with the right intentions is a habit, not a given. That's profound.

And then there's one more that feels incredibly relevant today: “It is the mark of an educated mind to be able to entertain a thought without accepting it.” We've lost that ability in modern discourse. If someone disagrees with you today, they're immediately your enemy. However, Aristotle encouraged people to think critically and openly—without fear of ideas.

Jacobsen: Any favourite Alexander the Great quotes?

Lyras: Absolutely. One of my favourite quotes from Alexander the Great comes from his sense of legacy. He spent a long time in the shadow of his father, King Philip, who had already conquered much of the region surrounding Macedonia. Alexander was deeply self-conscious about what he would leave behind.

He once said: “In the end, when it’s all over, all that matters is what you’ve done. All that matters is what you leave behind.” That was a driving force for him. He wanted a legacy, and he certainly achieved one—but it came from a deep-rooted need to surpass his father.

Jacobsen: Alex, thank you so much for your time today. I appreciate it.

Lyras: It was great to get into it with you. Looking forward to the next one.

Making Witch Hunting History: Dr. Leo Igwe's Fight for Justice

Dr. Leo Igwe & Scott Douglas Jacobsen



Nigerian human rights activist and lawyer Dr. Leo Igwe needs no introduction on these pages.

Dr. Leo Igwe and I [Scott Douglas Jacobsen] have more in common than a birthday (26 July); we both abhor human rights violations, mob justice, superstition, ritual murders, impunity and other forms of injustice.

I would like to take this opportunity to congratulate Dr. Leo Igwe on his relentless fight against superstition, witch hunting and ritual murders, and encourage him to continue to do so.

How can Nigeria's legal system be strengthened to effectively prosecute witchcraft-related abuses?

Dr. Leo Igwe is a Nigerian human rights advocate, scholar, and founder of the Advocacy for Alleged Witches (AfAW). With decades of activism, Igwe has dedicated his career to defending those falsely accused of witchcraft, combating superstition, and advancing secular human rights. He has partnered with international and national organizations to confront harmful practices rooted in fear and cultural beliefs, particularly targeting women, children, and people with disabilities. A vocal critic of religious extremism and media sensationalism, Igwe promotes critical thinking, education, and legal reform. His work stands at the intersection of grassroots advocacy, public enlightenment, and global humanism.

In this interview with Scott Douglas Jacobsen, Igwe intensified campaigns across Nigeria in 2025 to defend victims of witchcraft accusations. Through unprecedented collaborations with organizations such as the International Federation of Women Lawyers, the National Human Rights Commission, and disability rights groups, AfAW has expanded its outreach to over 15 states. Initiatives include memorial events, legal interventions, media engagement, and direct support for victims. Despite cultural and religious resistance, Igwe emphasizes that witchcraft is a myth, urging communities to shift from fear-driven persecution to rights-based advocacy. His work highlights growing momentum, though challenges remain entrenched.

Scott Douglas Jacobsen: Today, we are here with the prolific activist, Dr. Leo Igwe of Nigeria, founder of the Advocacy for Alleged Witches (AfAW). Our primary focus is advocacy for people accused of witchcraft. A lot has happened this year, and we can dive into some specific events because I have notes. In your view, what have been the most significant achievements so far?

Dr. Leo Igwe: One of the most significant developments this year is that we have organized more meetings and awareness programs than in any previous year since 2020. Even as I speak with you, I am in Port Harcourt, in Rivers State, where we are organizing an awareness event—an event to remember victims of witch hunts and ritual attacks. It is the first of its kind in the country and in the history of our campaign: victims are being remembered rather than demonized.

These victims are not being pre-judged as guilty or condemned. There has also been considerable interest from groups wanting to partner with us. We have seen unprecedented requests and welcoming gestures from different organizations and civil society groups. For instance, the International Federation of Women Lawyers (FIDA)—several state chapters—has reached out to co-organize events. Historically, their focus has been on women and children, and accusations of witchcraft were not central; that is changing as AfAW's work gains traction.

We have also engaged with the National Human Rights Commission of Nigeria (NHRC). Nigeria has 36 states and the Federal Capital Territory (Abuja), and some NHRC state offices are reaching out to co-organize events like the one we are holding on Saturday. They are ready to collaborate to highlight these abuses.

The Down Syndrome Foundation Nigeria has also contacted us to partner. They work on disability issues. Unfortunately, people with disabilities are often stigmatized or labelled as “possessed,” which leads to ostracism and harmful so-called “spiritual” interventions.

A recent example that drew national attention was a reported case in Calabar in February 2025, where a pastor allegedly killed his daughter, a child with Down syndrome, claiming she could transform into a snake. Cases like this show how superstition and stigma can turn deadly, and we are working with disability advocates to confront these beliefs and protect vulnerable families.

In terms of people who are accused, demonized, or stigmatized—whether because of disability or because of problems within the community—this has been a significant focus this year. We have now organized or collaborated in organizing events in over 15 states across Nigeria. By next week, we are planning an event in Niger State, in the north of the country. That will be the first event we have organized there, and we hope to use the opportunity to strengthen our partnerships with local groups and build a more robust mechanism for defending the accused.

That said, these collaborations do not come without challenges. For instance, in Niger State, we are partnering with women’s rights and children’s rights groups. They told us they would prefer not to have accused persons present, because their presence might trigger confrontation with accusers or with those who suspect them of being witches. This has been difficult, but we see it as a step toward educating people that everyone should stand as an advocate for the accused.

Many people still hold on to those beliefs and fears, even while showing some sympathy for the accused. However, sympathy is not enough. The accused are innocent. The law is on their side. So we want to find ways to reduce fear and anxiety and encourage communities to join us in openly and categorically supporting those accused of witchcraft.

Our meetings are not always characterized by unanimous support for advocacy on behalf of alleged witches. Sometimes, participants insist that witchcraft oppression is real. For example, at a recent meeting in Owerri, a pastor argued that witchcraft affliction must be addressed.

This is the contradiction we face. People say they oppose torture, killing, and persecution, but at the same time, they continue to insist witches exist. For us at AfAW, this is contradictory. If anyone claims people really are witches, then the burden is on them to prove it—to vindicate or exonerate those accused, rather than subject them to persecution.

Religion and culture also reinforce these challenges. Christianity, Islam, and Nollywood movies all perpetuate the belief that witchcraft and demonic possession are real. These institutions and cultural products continue to fuel the mindset that sustains witchcraft accusations.

[Editor’s Note: <https://en.wikipedia.org/wiki/Nollywood>]

In the churches and in the mosques, these harmful ideas are still being promoted. We are working to weaken the grip of these narratives on people's minds and to chip away at what I call "witchcraft evangelism." It does enormous damage and undermines our work. We also want people to recognize that Nollywood films and African movies are fiction, not fact.

The filmmakers reflect the myths and beliefs of society, but they are still telling stories, not recording reality. We want to help reorient society so that these movies are understood as cultural fiction. These are some of our successes, but also some of our challenges. Still, we see steady progress as more people begin to realize that something does not add up when it comes to witchcraft accusations. More groups are welcoming us and reaching out to cooperate, so that together we can address and dispel this phenomenon.

Jacobsen: Now, about specifics, in Owerri, Imo State, on September 2–3, we observed the International Day Against Witch Hunts. That was an event reaffirming material and psychosocial support. What was the big takeaway from that event this year?

Igwe: A lot. In Owerri, for the first time, we marched through the streets of the city, sharing flyers and speaking with people about the problem. We also visited the palace of the traditional ruler, Eze Clinton, who received us warmly and pledged his support to our campaign. That was an important milestone.

Another highlight was a presentation by our legal counsel, Mr. Okorie, on witchcraft accusations and the law. In Nigeria, accusing someone of witchcraft is a criminal offence. It is a form of criminal defamation, but most Nigerians are unaware of this—or if they are, they do not take it seriously, because their beliefs often outweigh what is written in the law. Mr. Okorie made it clear that even calling someone a witch can lead to prosecution. If this is done in a church or public gathering, the entire act is criminal.

He gave the example of a crusade organized in Imo State shortly after our event. The theme was "That Witch Must Die." We reported it to the police, who summoned the pastor, but unfortunately did not prosecute him. Mr. Okorie explained to our participants that such gatherings are legally actionable, and anyone who participates in them could also be held liable. His legal perspective shocked many people, as they were unaware that the law was so clear on this matter.

We also had some victims from different communities share their experiences, which reinforced the urgency of our campaign.

We also heard from victims who recounted their stories and experiences. One woman in particular, Mrs. Regina, told us that after some people in her family died, she was forced to undergo a ritual. They bathed the corpse, washed the body, and gave her the water to drink as an "exoneration" ritual. She is one of the people we are supporting now, trying to provide her with all the necessary help to get back on her feet.

Another experience I had was visiting a street named after a victim of ritual killing, Ikechukwu Okoroho, who was murdered about 30 years ago. A street was named in his memory. I went to that street and to the scene where he was killed, according to reports. These are some of the key takeaways from the Owerri, Imo State event.

Jacobsen: There was also a case intervention in Ebonyi State on August 20, involving the banishment of Joseph Agwu from Unwuhu community. The case called on the state to prosecute the attackers, compensate the victim, and end the practice of banishment. Could you elaborate on that specific case?

Igwe: Yes, Joseph's case is one of several in Ebonyi. He was accused of witchcraft and banished from his community. His property was destroyed, and he was forced to leave. We reached out to him, and he recounted his ordeal. We are appealing to the state authorities to step in and protect people like him.

Another successful intervention we made was in the case of Mr. Kingsley, who had also been accused. He was paraded through the streets, humiliated, and substances were poured over his body. When we got the information, we immediately contacted the police.

Thanks to that intervention, Kingsley is now back in his community. I met him recently, and he told me how happy and relieved he was. People now look at him with respect rather than the scorn he used to face. This was a real success story.

Of course, not all cases succeed. Sometimes incidents happen in rural communities where it is difficult for us to intervene. Accessing those areas can be dangerous—there are threats of beatings, mob attacks, or even killings. People in those communities often suspect that anyone investigating is there to help the police prosecute them. So yes, we have had some successes, but the challenges remain significant.

Jacobsen: There were also several roundtables. For example, in Ekiti State from August 19 to 21, there was a stakeholder roundtable aligned with the World Day Against Witch Hunts. There were also NHRC partnerships in Kano, Okoro, Ondo, and Yola, Adamawa. Across the year, there were several such meetings—on January 21, March 6, July 21, and August 19–21. What is the role of these roundtables, and what were the key takeaways from each?

Igwe: For the one we held in Yola early in the year, the big takeaway was that too often, when these cases are reported, nothing is done. They appear in the news and then disappear. Victims receive no help or support.

Since 2020, AfAW has been a game-changer. We step in on the side of the accused to support and empower them. In Yola, our message was clear: there is now an organization that stands for the accused. We introduced ourselves, explained what we do, and intervened in a specific

case where a parent and his partner tortured a girl to death. The mother had been accused of witchcraft, and the children were said to have “inherited” it from her. The girl was tortured and died. We have been working hard to support the mother and her three surviving children, and to push for justice.

That was our first meeting in Yola, and like with many of these events, participants told us nobody else was doing what we are doing. We know why—few people have the conviction and understanding that we at AfAW bring. However, we made it clear there is now a place where the accused can seek support, and an organization keeping watch on these cases. That was our takeaway from Yola.

In Ondo, we also held an event and combined it with a radio program. A woman named Olaemi Ijogun attended after hearing us on the radio. She told us how she had been accused as a child and beaten. Her case was heartbreaking. She said that both she and her sister had been accused of being initiated into a coven when they were very young.

In Olaemi’s case, the accusations came from a relative who claimed to have seen her and her sister in a dream. The parents were told the girls were going to covens at night. As a result, they were not allowed to sleep. They were forced to kneel and raise their hands through the night because the parents believed that if they slept, they would travel spiritually to the coven. The girls were denied sleep for several nights.

The stigma followed Olaemi to school, where it negatively impacted her social life. She still breaks down when recounting the trauma, which she did at our event. She called on people to stop making accusations because they leave an indelible mark on the minds and psyches of children. Since then, she has been working with us to advocate against witchcraft accusations.

For instance, she joined us in Ekiti State during the World Day Against Witch Hunts event. There, we encountered a case where a 10-year-old girl accused her grandmother of initiating her into a coven and of spiritually murdering people. This accusation was made on the radio after a station invited the family to speak. As a result, the grandmother’s business collapsed, and she was ostracized; the community avoided her. We intervened to reassure her that she had no hand in such things.

The background is that the family’s youngest child, about two years old, had been sick since birth. The grandmother was blamed for the illness. When I interviewed the mother of the 10-year-old, she even told me that the grandmother had “taken away the intelligence” of the children, causing them to do poorly in school, and was also responsible for the family’s financial struggles. In other words, they blamed the grandmother for virtually every problem.

To address this, we provided the family with money to conduct a medical test on the child, so we can determine the real medical problem and treat it appropriately. This shows that we are not only holding events, but also taking practical steps to intervene. We extend solidarity by

combining advocacy with direct support. We are helping the grandmother, the victim of the accusation, while also ensuring that the sick child receives medical treatment. These are some of the key outcomes from the Ekiti State event.

Jacobsen: How did the World Day Against Witch Hunts itself go?

Igwe: It was observed on August 10. That year it fell on a Sunday. In Nigeria, the best thing you can do on a Sunday is either go to church or stay at home. Suppose you organize anything else on that day. In that case, it is not likely to attract much participation—except for the few atheists and humanists in the country.

On August 10, the World Day Against Witch Hunts, I attended a church where the pastor regularly preaches against witch hunting. In our work, we identify religious leaders who speak out against these practices. It is not easy, of course, but we make every effort to find such churches. I was told about this one, contacted the pastor, and he confirmed that he preaches against witch hunting. So I went there to listen to his sermon. We also recorded it so that we could use it later to show other churches that this kind of preaching is possible and necessary.

It was a small church, with maybe 50 participants—tiny compared to the massive congregations you see in Nigeria, where tens or even hundreds of thousands gather. That probably explains why this church holds what you might call a minority position in the religious landscape. Still, that was where I spent the day.

Before and after August 10, we have continued organizing events in various states to remember victims of witch hunts and ritual attacks. It has gone well. People are coming out and saying, “At last, there is a space where we can feel vindicated, where we can share our stories in front of an audience that supports us, rather than seeing us as guilty.” That has been the spirit of these gatherings. In fact, we could not accommodate all the events in August, which is why some of them were pushed into September. For us at AfAW, the World Day Against Witch Hunts has not really ended. Our event this Saturday will conclude this year’s cycle of activities tied to that observance.

Jacobsen: Let us turn to the media side of things—ongoing public education, advocacy, op-eds, and briefings. Which news and opinion publications have been most effective in disseminating information about this campaign, the organization, and the harm caused by these superstitions?

Igwe: We have had coverage of our activities in several online and mainstream media outlets. Some journalists have even drawn our attention to cases in which we later intervened. Among Nigerian media organizations, I must mention Sahara Reporters, *This Day*, and The Eagle Online, which have been supportive.

We have also had coverage in other outlets, such as the *Nigerian Tribune*, *Punch*, and *The Sun*. Some of these online and print organizations have tried to highlight the work we are doing.

However, let me be clear—before now, media agencies have overwhelmingly been part of the problem. Their reporting on witchcraft accusations often reinforces the very narratives we are trying to dismantle. This is something I consistently point out to them during media interactions.

Many journalists still report accusations in sensational ways. They tell me the more spectacular, the better—for clicks and traffic. They call it “clickbait.” So, you see headlines like “Witch Crash-Lands” or “Bird-Woman Found in Village.” It is absolute nonsense, but it generates attention. Moreover, in their pursuit of attention, they misinform the public, mislead communities, and do real harm.

These reports are unprofessional and unethical. Journalism should be about reporting facts, and it should be balanced. Instead, in their quest for traffic, media houses end up endangering lives. For example, there was a radio program where a child accused her grandmother of initiating her into witchcraft. We intervened, and when we left, the station manager admitted to me, “Leo, it was this radio program that caused the problem.” He realized it had put an innocent woman in danger and destroyed her socially.

So yes, the media have been part of the problem. However, with the kind of engagement we are doing at the Advocacy for Alleged Witches (AfAW), some outlets are beginning to rethink. Some are realigning and realizing just how unprofessional and unethical their reporting has been. They are slowly starting to highlight our perspective. However, we still have a long way to go. Nigerian media organizations still thrive on sensationalism.

The media still thrives on sensational headlines—stories designed to attract attention and appeal to primitive superstitions that people find exciting. Slowly and steadily, some outlets are beginning to support what we are doing. However, there is still a tremendous amount of work ahead.

Another challenge is this: while media agencies are quick to publish sensational, false, and misleading reports about witchcraft—often for free—when we want to put forward our perspective, they demand large sums of money. Both online and broadcast outlets do this.

For example, if we want to appear on television, they charge between \$500 and \$1,000 just for the appearance. Additionally, you may need to travel, pay for flights, and cover accommodation costs. This makes enlightenment and advocacy extremely expensive, even though it is precisely what the country needs to counter these harmful narratives.

Jacobsen: Do you have any final points on that last topic?

Igwe: Yes, while a few media organizations are beginning to report witchcraft accusations more responsibly—rather than treating witchcraft itself as a fact or as a “certified” part of African culture—the progress is limited. Some outlets are starting to understand AfAW’s position and provide more balance. However, we are still far from the cultural shift we need. That kind of change will not happen through one report or even one event. It requires intensive public education and sustained enlightenment.

Unfortunately, in this area, many media stations have not been supportive. They are quick to publish sensational stories, like “an elderly woman turned into a bird” or “a witch crash-landed on her way to a meeting,” as was recently reported in Delta State. These kinds of stories get free publicity.

However, when AfAW attempts to purchase airtime to educate the public, we encounter significant costs. Media outlets charge us considerable amounts of money, making enlightenment campaigns very expensive. The imbalance is stark: free space for superstition, but costly barriers for rational education.

Meanwhile, churches and religious organizations that actively promote witchcraft narratives are given abundant airtime. They advertise events with themes like “That Witch Must Die” or “Exposing the Mysteries of Witchcraft.” These programs receive free promotion, which reinforces harmful beliefs.

By contrast, when we present our position—saying plainly that witchcraft is a myth—we are given little space, asked to pay heavily, and sometimes even put under pressure during media interviews. The pressure is on us to “prove” that something imaginary does not exist, instead of challenging those who claim it does.

The media landscape is still heavily skewed toward reinforcing witchcraft beliefs. We have not yet reached the paradigm shift where media establishments themselves start questioning and dismantling these narratives. That remains the challenge before us.

The cultural shift we need will only come when the media itself transforms. Until then, they will not welcome our programs in the way they should. Even when we pay for airtime, they often schedule us in the middle of the day, when people are busy at work. They refuse to give us prime slots in the evening or late at night—times when churches preach about witchcraft to audiences at home around the dinner table.

Without media on our side, we cannot fully succeed in making witch-hunting history in this region. That is why this work is so critical.

Jacobsen: There was a memorial action on August 29, connected to victims of ritual killings. You visited a hotel site linked to one of those incidents, to connect memory with today's anti-witch hunt work. Could you explain what happened at that hotel, and how many victims are we talking about?

Igwe: I visited because of the incident that happened there in September 1996, almost 29 years ago. What happened then is still happening today. For example, earlier this year, in February 2025, in Lagos, a young man murdered his girlfriend, used an axe to break her head, and drained her blood into a calabash, supposedly for rituals. That case mirrors what happened at the Otokoto Hotel in 1996.

At Otokoto, the victim was an 11-year-old boy who sold peanuts on the streets. A hotel gardener lured him inside, gave him a drugged drink, and when the boy became unconscious, he cut off his head. The man was apprehended while attempting to deliver the head to someone who had ordered it for ritual purposes.

The news caused a massive uproar. There were riots in the city, and people began burning the houses of those suspected of being involved.

The people labelled as “ritualists,” in other words, those involved in ritual syndicates or racketeering, were the focus of that uproar. My visit to the Otokoto Hotel aimed to remind the people of Imo State that this practice has been ongoing for far too long and must come to an end.

The government seized the hotel property, and today it is used by the police. Not far from the police station, there is a street named after the young boy who was murdered. Those responsible were eventually arrested, and some received life imprisonment while others were sentenced to death.

I visited that property to show that the same problem we saw nearly three decades ago is still with us today—only in new forms. Now, people kill their girlfriends, relatives, or acquaintances for what they call “organ harvesting.” They believe specific organs can be used in rituals to produce wealth, success, or power.

The narratives of religion, miracles, magic, and supernatural intervention fuel these beliefs. All of them reinforce the idea that ritual killings can deliver prosperity. What we are confronting is a Herculean task—a complex, many-headed monster of superstition and fear. Only the flame of reason, compassion, critical thinking, and skeptical inquiry can provide hope for society and for the victims.

Jacobsen: Thank you for the opportunity and your time, Leo.

Black Hole Shadows & Lorentz Violation: Dr. İzzet Sakallı on EHT Tests, Wormhole Mimickers, and Tsallis-AdS Phase Transitions

Dr. İzzet Sakallı & Scott Douglas Jacobsen



Abstract

This interview with Dr. İzzet Sakallı addresses observational and theoretical strategies for testing beyond-general relativity physics using black-hole imaging, lensing, and gravitational-wave data. Sakallı outlines how Event Horizon Telescope (EHT) shadow observations—motivated by horizon-scale images of M87* and Sagittarius A*—can be used to probe Lorentz-invariance-violating photon propagation while accounting for plasma and accretion-flow systematics through multiwavelength, polarization, timing, and variability diagnostics. He then describes limitations of applying Gauss–Bonnet-based weak-deflection methods in geometries with torsion, emphasizing non-unique extensions of curvature invariants and ambiguities in photon trajectories and boundary terms. Sakallı further identifies lensing and multimessenger signatures that could distinguish thin-shell wormholes from black holes, including gravitational-wave echo phenomenology and nonstandard shadow and microlensing

features, noting that echo searches have placed constraints in existing data. The final section considers whether information-theoretic diagnostics (e.g., Fisher information and relative-entropy measures) can constrain quantum-gravity–motivated parameters at current sensitivities, and discusses how non-extensive (Tsallis) entropy may alter Anti-de Sitter black-hole phase structure and interpretations of the Hawking–Page transition.

Keywords

Accretion-flow systematics, Anti-de Sitter (AdS) black holes, Boundary terms, Catechism (contextual mention), Chromatic shadow tests, Dispersion, Einstein–dilaton–Gauss–Bonnet gravity, Event Horizon Telescope (EHT), Faraday rotation, Fisher information, Gauss–Bonnet term, Gravitational-wave echoes, Hawking–Page transition, Information theory, Kullback–Leibler divergence, Lens time delays, Lorentz invariance violation, Microlensing, Multiwavelength imaging, Mutual information, Photon ring, Polarization decomposition, Quantum-gravity phenomenology, Ray tracing, Relative entropy, Ringdown, Thin-shell wormholes, Torsion, Tsallis entropy, Variability monitoring

Introduction

Horizon-scale very-long-baseline interferometry has made black-hole “shadow” observables empirically accessible, with EHT results providing resolved ring-like emission structures around M87* and Sagittarius A*. These measurements create opportunities to test extensions of general relativity and related high-energy frameworks using imaging, polarization, and time-domain signatures, while confronting the dominant astrophysical systematics introduced by hot, magnetized plasma in the accretion flow.

Within this context, the interview explores four linked problem areas: (1) discriminants for Lorentz-violating backgrounds in the presence of plasma effects; (2) the applicability limits of Gauss–Bonnet approaches to weak deflection when torsion is present; (3) observational signatures that could exclude thin-shell wormholes as black-hole mimics; and (4) whether information-theoretic and non-extensive-thermodynamics formalisms provide useful constraints at current or near-future sensitivities. The discussion connects these themes to established theoretical landmarks, including the Hawking–Page transition in AdS thermodynamics and the non-extensive entropy framework introduced by Tsallis.

Professor Izzet Sakallı is a theoretical physicist at Eastern Mediterranean University whose research bridges quantum mechanics, general relativity, and observational astronomy. With over 180 publications exploring black hole thermodynamics, modified gravity theories, and quantum corrections to spacetime, his work sits at the exciting frontier where abstract mathematics meets observable reality. In this interview, he discusses the challenges of testing exotic gravity theories, the quest to observe quantum effects in astrophysical systems, and what the next generation of telescopes and gravitational wave detectors might reveal about the quantum nature of spacetime.

Scott Douglas Jacobsen: Which shadow features best test Lorentz-violating backgrounds once plasma and accretion-flow systematics are included?

Professor Izzet Sakali: The Event Horizon Telescope's stunning images of M87* and Sagittarius A* opened an unprecedented window for testing fundamental physics. These images show a bright ring of emission surrounding a dark shadow—the gravitational silhouette of the black hole. But extracting constraints on exotic physics requires carefully separating genuine signatures of modified spacetime from astrophysical contamination.

Lorentz invariance—the principle that physical laws don't depend on your orientation or velocity—is a cornerstone of both special and general relativity. Various approaches to quantum gravity predict potential violations at high energies or in strong gravitational fields. These violations would modify how photons propagate, potentially changing the observed shadow.

The challenge is that accretion disks—the hot, ionized material spiraling into black holes—are messy astrophysical environments. The plasma causes Faraday rotation of polarized light, introduces frequency-dependent delays through dispersion, absorbs some wavelengths while transmitting others, and Doppler shifts due to its orbital motion. All these effects must be modeled and subtracted to reveal underlying spacetime properties.

The most robust tests use signatures that depend differently on fundamental physics versus astrophysical processes. Multi-wavelength shadow measurements provide one such discriminator. If Lorentz violation affects photon propagation, the shadow size should vary with observing frequency in a specific pattern—different from how plasma effects scale. Observing at 230 GHz, 345 GHz, and potentially infrared wavelengths with the next-generation Event Horizon Telescope would allow separating these effects.

The photon ring—actually a series of nested subrings from photons completing multiple orbits before escaping—offers another powerful probe. The time delay between successive subrings depends on geodesic motion in the spacetime geometry, largely insensitive to accretion flow details since it's a geometric effect. Lorentz violation would modify these time delays in characteristic ways.

Polarization patterns provide additional leverage. While plasma Faraday rotation certainly affects polarization, Lorentz violation can break azimuthal symmetry in distinctive ways. Decomposing the polarization pattern into angular components helps separate astrophysical rotation (which affects all components similarly) from symmetry-breaking physics.

Time variability studies offer yet another approach. Monitoring shadow features over days to months separates quasi-periodic variability from orbital motion, flares and turbulence in the accretion flow, and any secular trends from fundamental physics. If the shadow properties slowly drift in ways inconsistent with astrophysical explanations, that might signal Lorentz violation.

Current EHT observations constrain deviations from general relativity at the ten to twenty percent level. This translates to saying that if Lorentz violation exists, its characteristic energy scale must be at least one or two percent of the Planck energy—still far below the Planck scale but better constraints than many other tests achieve.

Future improvements will come from multiple directions. The next-generation EHT adds more telescopes and space-based stations, increasing angular resolution. Higher observing frequencies probe smaller scales. Broader bandwidth improves sensitivity. Perhaps most importantly, coordinated multi-messenger observations—combining radio interferometry with X-ray timing, optical monitoring, and potentially gravitational waves if the black hole is in a binary—would provide complementary information less susceptible to systematic uncertainties.

Jacobsen: What are the limits of the Gauss-Bonnet approach for weak deflection in spacetimes with torsion?

Sakalli: Einstein-dilaton-Gauss-Bonnet gravity represents an important modification of general relativity emerging from string theory. The Gauss-Bonnet term involves products of curvature quantities in a specific combination that's topologically interesting. In standard four-dimensional spacetime, this term is actually a topological invariant—it doesn't affect the equations of motion by itself. But when coupled to a dilaton field (a scalar field from string theory), it produces genuine modifications to gravity.

These modifications become significant in strong-field regions near black holes or in the early universe. For weak gravitational fields—like sunlight grazing the Sun—the corrections are typically small but calculable. We can expand solutions perturbatively, treating the Gauss Bonnet term as a small correction to general relativity, and calculate how it affects phenomena like light deflection. However, this approach encounters fundamental difficulties when spacetime possesses torsion.

Torsion represents a twisting of spacetime, a type of geometric structure absent in general relativity but present in some extended theories. Einstein-Cartan theory, for instance, allows spacetime to have both curvature and torsion, with torsion sourced by intrinsic spin of matter.

The difficulty is that the Gauss-Bonnet term is defined for Riemannian geometry—geometry with curvature but no torsion. Generalizing it to spaces with torsion isn't unique. Multiple inequivalent extensions exist: the Nieh-Yan form, modifications involving the Pontrjagin term, generalizations of the Holst term. These different extensions give different physics, and there's no obvious principle selecting one over others.

Furthermore, torsion couples to matter fields differently than curvature. For photons—massless spin-one particles—the interaction with torsion is subtle. Traditional geodesic equations must be reconsidered. The very notion of "geodesic" becomes ambiguous because the connection determining parallel transport includes torsion contributions.

The weak-field expansion faces technical breakdowns in torsion backgrounds. We typically expand the metric as small perturbations around flat spacetime. But when both curvature and torsion are present, the perturbative hierarchy becomes unclear. If torsion is as large as curvature perturbations, our expansion scheme breaks down.

There are also boundary term subtleties. In formulating gravitational theories variationally, boundary terms matter for defining conserved quantities and obtaining correct equations of motion. The Gibbons-Hawking-York boundary term supplements the Einstein-Hilbert action in general relativity. With torsion, additional boundary terms are needed, and they affect asymptotic quantities crucial for calculating observable effects like deflection angles.

Some torsion-Gauss-Bonnet couplings lead to pathologies: faster-than-light propagation, violations of causality, or violations of energy conditions guaranteeing stability. These signal the theory's breakdown rather than providing viable alternatives to general relativity.

Observationally, constraints on torsion in astrophysical contexts are weak. Particle physics experiments provide stronger bounds from spin-dependent effects. But astrophysical torsion contributions must satisfy stringent limits, roughly one part in a billion billion per centimeter from precision tests.

For analyzing light deflection in torsion-rich spacetimes, alternative approaches work better than weak-field expansions. Full numerical ray-tracing through exact solutions avoids perturbative ambiguities. The Newman-Penrose formalism, adapted to non-Riemannian geometry, provides another rigorous framework. Modified Shapiro time delays offer complementary probes less sensitive to expansion scheme choices.

Jacobsen: Which lensing signatures would rule out thin-shell wormholes for compact black-hole mimics?

Sakalli: Wormholes—hypothetical tunnels connecting distant regions of spacetime—have captivated physicists and science fiction writers for decades. While the Schwarzschild solution allows mathematical wormholes, they're unstable and not traversable. Maintaining a stable, traversable wormhole requires exotic matter violating energy conditions, specifically matter with negative energy density.

Thin-shell wormholes represent a particular construction where exotic matter is confined to a narrow region around the wormhole throat, minimizing the total amount of energy-condition violating material needed. From a distance, such objects might mimic black holes—appearing as compact, massive objects with strong gravitational fields. Distinguishing them observationally is crucial, as finding a wormhole would revolutionize physics.

Several observational signatures could definitively distinguish wormholes from black holes. The most dramatic is gravitational wave echoes. When a gravitational wave rings down after a black hole merger, the signal decays smoothly—the perturbations fall into the singularity. But if the

object is a wormhole, perturbations can traverse the throat, reflect off the geometry on the far side, return, and emerge again. This creates periodic echoes in the gravitational wave signal, with the time delay determined by the wormhole's size and geometry.

Detecting echoes is challenging. The reflected signal is weaker than the initial ringdown, requiring high signal-to-noise ratio observations. Current searches in LIGO/Virgo data have found no convincing echoes, placing limits on how common wormholes might be if they exist. Future detectors with better sensitivity, particularly Einstein Telescope and Cosmic Explorer, should significantly improve echo searches.

The shadow morphology offers another discriminator. Black holes cast circular (or slightly elliptical for rotating ones) shadows. Wormholes can produce more exotic shadow shapes: non simply-connected topology (hole within hole), asymmetric brightness from different light paths traversing the throat, or chromatic effects if the throat has frequency-dependent properties. The Event Horizon Telescope's resolution is reaching the point where such features might be detectable for nearby supermassive objects.

Microlensing provides a third signature. When a compact object passes in front of a background star, its gravity acts as a lens, magnifying and distorting the star's image. Black holes create characteristic magnification curves. Wormholes add complexity: light paths traversing the throat create additional magnification spikes, producing asymmetric light curves distinct from black hole lensing. Large microlensing surveys searching for dark matter or planets could potentially detect such signatures if wormholes exist in our galaxy.

Time delay distributions between multiple images offer another probe. Gravitational lenses create multiple images of background objects arriving at different times. The distribution of these delays depends on the lens's mass profile. Wormholes' different internal geometry would create distinctive delay patterns.

Future interferometric observations using nulling techniques could enhance faint photon ring structure while suppressing the bright accretion disk. Wormholes' multiple light traversal paths would create interference patterns distinct from black holes.

The theoretical challenge is that thin-shell wormholes face severe difficulties. Most solutions are dynamically unstable—perturbations cause them to collapse or expand. The exotic matter requirements are enormous, far exceeding what quantum field theory allows in normal circumstances. No known astrophysical process forms wormholes, unlike black holes which form from stellar collapse.

Nevertheless, thoroughly testing whether astrophysical compact objects might be horizonless alternatives to black holes is scientifically important. General relativity so strongly predicts black holes that finding something else would indicate either a fundamental flaw in our understanding or exotic physics beyond the standard model. The observational tests described above will systematically close loopholes, either confirming black holes or revealing surprises.

Jacobsen: Can information-theoretic diagnostics reveal quantum gravity corrections at current observational sensitivities?

Sakalli: Information theory—the mathematical framework for quantifying information, uncertainty, and correlations—provides a novel lens for exploring quantum gravity. Traditional approaches focus on measuring physical quantities like masses, spins, and frequencies. Information-theoretic approaches ask: how much information do observations carry about the underlying physics? Can we detect quantum gravitational effects through information content rather than direct parameter measurements?

The holographic principle suggests that information in any region of space is bounded by the region's surface area rather than volume—as if the three-dimensional world is holographically encoded on a two-dimensional boundary. For black holes, this manifests as the Bekenstein-Hawking entropy being proportional to horizon area. Quantum corrections modify this relationship, adding logarithmic and higher-order terms in the area.

One information-theoretic approach analyzes the entropy of black hole shadows. The shadow pattern observed by telescopes carries information about the spacetime geometry. We can quantify this through Shannon entropy: treating the brightness distribution as a probability distribution and computing its entropy. Quantum corrections alter how photons propagate near the horizon, subtly changing the shadow structure and therefore its information content.

The challenge is scale. For astrophysical black holes, Planck-scale quantum corrections are suppressed by the ratio of the Planck length to the black hole size—roughly 10 to the minus 78th power for a solar-mass black hole. Direct detection of such tiny effects is hopeless with foreseeable technology.

However, indirect signatures might be more accessible. Rather than looking for tiny shifts in individual measurements, we can examine correlations and information flow. For gravitational wave signals, the mutual information between early inspiral and late ringdown phases quantifies how much information from the initial state survives to the final state. Quantum corrections introduce non-Markovian effects—memory effects where the system's evolution depends on its history rather than just its current state. These memory effects could create measurable correlation patterns.

Fisher information quantifies how much information data carries about physical parameters. For Event Horizon Telescope observations, we can construct the Fisher information matrix describing precision limits on measuring black hole mass, spin, and potential quantum correction parameters. Current analysis suggests achieving roughly 10 to 30 percent precision on dimensionless quantum parameters—not sufficient to detect Planck-scale effects but potentially constraining if quantum gravity involves larger characteristic scales.

Relative entropy (also called Kullback-Leibler divergence) measures how much observed distributions differ from general relativity predictions. Accumulating this measure across many

observations provides a statistical test: are we seeing what general relativity predicts, or are systematic deviations emerging?

Population statistics offer multiplicative power. While individual gravitational wave detections have limited precision, combining information from hundreds or thousands of events (expected in the next decade) increases statistical power. If quantum corrections produce consistent small biases across all events, population-level analysis might reveal them.

Cross-correlation approaches combine complementary observations. Measuring a black hole's mass from gravitational waves and independently from its shadow size provides a consistency test. If quantum corrections affect these measurements differently, inconsistencies would signal new physics even if neither measurement alone shows deviations.

For primordial black holes—hypothetical small black holes from the early universe—information theoretic signatures might be more accessible. Their much smaller sizes enhance quantum corrections substantially. If primordial black holes exist and their evaporation products are detected, the information content in their emission spectra could constrain quantum gravity.

Realistic assessment requires acknowledging limitations. Directly detecting Planck-scale quantum gravity through astrophysical observations probably remains beyond reach. However, information-theoretic methods might reveal emergent quantum phenomena accumulating at astrophysical scales, modified information flow during black hole evaporation, or indirect constraints ruling out large classes of quantum gravity theories. These indirect routes may prove more fruitful than searching for minute corrections to individual measurements.

Jacobsen: How do non-extensive entropy formalisms reshape Anti-de Sitter black hole phase diagrams?

Sakalli: Non-extensive statistical mechanics, particularly Tsallis entropy, offers a fascinating generalization of conventional thermodynamics that has profound implications for black hole physics. The standard Boltzmann-Gibbs entropy assumes that systems are extensive—meaning the entropy of two independent systems simply adds. But for systems with long-range interactions, like gravity, or systems with fractal structure, this additivity breaks down. Tsallis introduced a parameter, typically called q , that quantifies this non-extensivity. When q equals one, we recover standard thermodynamics; deviations from unity signal non-extensive behavior.

For black holes in Anti-de Sitter spacetime—a universe with negative cosmological constant that curves like a saddle rather than a sphere—thermodynamics becomes particularly rich. AdS black holes can undergo phase transitions remarkably similar to everyday substances transitioning between solid, liquid, and gas phases. The famous Hawking-Page transition, where thermal radiation in empty AdS competes with forming a black hole, parallels water freezing into ice.

When we replace standard entropy with Tsallis entropy, the phase diagram transforms dramatically. The temperature-entropy relationship changes because temperature, defined as the derivative of energy with respect to entropy, now involves the non-extensivity parameter. For astrophysical black holes with enormous entropy, even small deviations of q from unity can shift transition temperatures substantially.

The heat capacity—which determines thermodynamic stability—becomes modified in intricate ways. Standard AdS black holes have a critical radius where heat capacity diverges, signaling a phase transition. With Tsallis entropy, this critical point shifts. Depending on whether q exceeds or falls below unity, the transition can occur at smaller or larger radii, or in extreme cases, disappear entirely or split into multiple transitions.

Perhaps most intriguing is the emergence of reentrant phase transitions—phenomena where increasing temperature causes the system to cycle through the same phase multiple times. Imagine heating ice, which melts to water, but continuing to heat causes it to refreeze, then melt again at even higher temperatures. Such behavior, absent in standard thermodynamics, appears naturally when black hole entropy becomes non-extensive. This suggests the underlying quantum gravity degrees of freedom organizing the horizon might have complex, fractal-like structure.

The Hawking-Page transition also shifts under non-extensive statistics. This transition represents a competition between entropy favoring thermal radiation spread throughout space and energy minimization favoring localized black holes. With modified entropy, the balance point changes. For q greater than one, transitions occur at lower temperatures; for q less than one, higher temperatures are needed.

Through the AdS/CFT correspondence—the remarkable duality between gravity in AdS space time and quantum field theory on its boundary—modifications to bulk thermodynamics reflect in boundary physics. The confining-deconfining transition in strongly coupled gauge theories, relevant for understanding quark-gluon plasma in heavy-ion collisions, would exhibit modified behavior if the gravitational dual involves non-extensive entropy.

The physical origin of non-extensivity in black holes remains debated. It might arise from quantum fluctuations of the horizon, long-range gravitational correlations between horizon degrees of freedom, or fractal microstructure of spacetime near the Planck scale. Observationally, constraining the parameter q from astrophysical black holes is extremely challenging, but laboratory analogs using cold atoms or condensed matter systems might provide testable predictions.

Jacobsen: What about the interpretation of Hawking-Page transitions under non-extensive entropy?

Sakalli: The Hawking-Page transition represents one of the most elegant connections between quantum field theory, gravity, and thermodynamics. In pure AdS spacetime, we can have

thermal radiation at some temperature, or we can have a black hole at that temperature. Which configuration has lower free energy depends on the temperature. At low temperatures, thermal radiation dominates; at high temperatures, black holes are favored. The transition between these phases occurs at a specific critical temperature.

When entropy becomes non-extensive, this picture enriches considerably. The free energy—energy minus temperature times entropy—depends on how entropy scales. With Tsallis entropy, the relationship between physical temperature and thermodynamic temperature becomes modified by a function of the horizon area. This modifies the free energy comparison between phases.

The latent heat—energy exchanged during the phase transition—changes dramatically. For standard black holes, the latent heat reflects the entropy jump when thermal radiation condenses into a black hole. With non-extensive entropy, this jump scales differently with black hole size. For large black holes and q greater than unity, the latent heat can become orders of magnitude larger than in standard thermodynamics, suggesting the transition involves reorganizing vastly more microscopic degrees of freedom.

The order of the transition can even change. Standard Hawking-Page is first-order, with discontinuous entropy at the transition point, like ice melting to water. But for certain special values of the non-extensivity parameter, the transition can become second-order, with continuous entropy but divergent heat capacity, like the magnetic transition in iron when heated above its Curie temperature. Between these regimes lie tricritical points where the transition character changes.

Through AdS/CFT, the Hawking-Page transition corresponds to confinement-deconfinement in the boundary gauge theory. At low temperatures, quarks and gluons are confined into hadrons—thermal AdS. At high temperatures, they form quark-gluon plasma—the black hole phase. Non-extensive modifications suggest the strongly coupled plasma might have non standard statistical properties, potentially observable in heavy-ion collisions at RHIC or LHC.

The interpretation becomes particularly interesting when considering the transition's dynamical aspects. How quickly does thermal radiation condense into a black hole? How long does the mixed phase persist? Non-extensive statistics introduce memory effects—the system's evolution depends on its history, not just its current state. This non-Markovian character might leave signatures in gravitational wave observations if black holes form dynamically through such transitions in the early universe.

Multiple phase transitions can emerge in extended phase space where we vary not just temperature but also pressure, charge, and angular momentum. The resulting phase diagrams can exhibit multiple critical points, isolated regions of stability, and complex connectivity between phases—far richer than standard thermodynamics allows.

Discussion

Across topics, Sakallı's central methodological emphasis is separability: identifying observables whose dependence on fundamental-physics modifications differs from their dependence on plasma, turbulence, and geometric degeneracies. For shadow-based Lorentz tests, the interview highlights multiwavelength scaling, polarization symmetries, and time-delay structure in near-horizon photon trajectories as candidate discriminants, conceptually aligned with theoretical work on photon-ring/subring structure and its geometric imprint.

For torsion-inclusive extensions, Sakallı frames the main limitation as non-uniqueness: Gauss–Bonnet constructions are straightforward in torsion-free (Riemannian) settings, but become ambiguous when generalized, complicating weak-field expansions and the definition of photon paths and conserved quantities. In this framing, robust inference favors approaches that minimize dependence on expansion choices, such as numerical ray tracing or formalisms explicitly adapted to non-Riemannian geometry.

Regarding wormholes, Sakallı treats “ruling out” as an empirical question tied to distinctive multipath propagation. He identifies gravitational-wave echo phenomenology and nonstandard lensing/shadow morphology as candidate falsifiers, while noting that echo searches have thus far reported null or constrained results in existing observational catalogs.

Finally, the interview positions information-theoretic diagnostics and non-extensive entropy as meta-tools: methods to quantify distinguishability between models and to explore how modified microphysical assumptions reshape thermodynamic phase structure. The interview's discussion of Tsallis entropy and its parameter q is grounded in the established non-extensive framework, and its application is linked to AdS black-hole phase behavior and the Hawking–Page transition as a benchmark for interpreting bulk/boundary thermodynamic changes.

Methods

The interview was conducted via typed questions—with explicit consent—for review, and curation. This process complied with applicable data protection laws, including the California Consumer Privacy Act (CCPA), Canada's Personal Information Protection and Electronic Documents Act (PIPEDA), and Europe's General Data Protection Regulation (GDPR), i.e., recordings if any were stored securely, retained only as needed, and deleted upon request, as well in accordance with Federal Trade Commission (FTC) and Advertising Standards Canada guidelines.

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This interview was conducted as part of a broader quantum cosmology book project. The responses reflect my current research perspective as of October 2025, informed by over 180 publications and ongoing collaborations with researchers worldwide.

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Mario Carlos Rocca on Ultradistributions, Ultrahyperfunctions, and Rigorous Quantum Field Theory in Einstein Gravity

Mario Carlos Rocca & Scott Douglas Jacobsen



Abstract

This interview with Mario Carlos Rocca addresses the definition and use of tempered ultradistributions and ultradistributions of exponential type within quantum-field-theoretic constructions. Rocca describes a framework in which tempered ultradistributions are continuous linear functionals on a space of entire test functions, and he outlines an analytic representation using contour integrals outside strips containing singularities. He explains why a naïve convolution formula does not generally exist for arbitrary pairs of ultradistributions, and presents a regulated construction using a complex parameter (λ), analytic continuation, and extraction of the λ -independent term to define convolution. The interview also summarizes how products of distributions of exponential type are handled via Fourier-transform relations and notes that associativity is not guaranteed in general due to the algebraic structure described as a ring with zero divisors. Rocca provides explicit finite convolution expressions for Wheeler propagators in massless and complex-mass cases and comments on microcausality, loop finiteness claims in the ultradistribution/ultrahyperfunction setting, and extensions from Minkowski space to semi-Riemannian and globally hyperbolic spacetimes. The final portion includes Rocca's high-level descriptions of Einstein gravity and Gupta–Feynman quantization, as well as formulas for graviton self-energy calculations presented in BTZ-background studies.

Keywords

Analytic continuation, Analytic representation, Associativity, BTZ gravity, Cauchy integral formula, Complex delta function, Complex mass, Convolution, Entire test functions, Einstein gravity, Exponential growth bounds, Fourier transform, Functional analysis, Gauge conditions, Gel'fand triplet, Ghost avoidance, Graviton self-energy, Gupta–Bleuler method, Gupta–Feynman quantization, Loop integrals, Microcausality, Minkowski space, Nuclear spaces, Operator-valued distributions, Propagators, Rigged Hilbert space, Schwartz distributions, Tempered ultradistributions, Ultradistributions of exponential type, Ultrahyperfunctions, Wheeler propagator, Zero divisors

Introduction

The interview concerns mathematical structures used in quantum field theory that generalize Schwartz distributions, focusing on tempered ultradistributions and ultradistributions of exponential type. In the interview text, these objects are presented through test-function spaces built from entire analytic functions with specified growth bounds, and through the rigged Hilbert

space (Gel'fand triplet) approach in which distributions are realized as continuous linear functionals on a nuclear test-function space. The discussion emphasizes analytic representations of generalized functions by contour integrals in the complex plane, where the contour is chosen to avoid bands containing singularities. The interview also treats the technical problem of defining convolutions and products for generalized functions: Rocca explains that direct convolution expressions are not always well-defined, motivating the use of regulators, analytic continuation, and a prescription selecting a parameter-independent term to define a convolution in cases of interest.

In addition to these core constructions, the interview addresses downstream physics-facing topics: conditions under which commutators satisfy microcausality in the ultrahyperfunction setting, claims about the treatment (or absence) of singularities within that formalism, and how these methods are applied to propagators, loop integrals, and model calculations in gravitational settings. These topics are framed in the interview by reference to established distribution theory traditions associated with Schwartz and to ultradistribution/ultrahyperfunction developments associated with Sebastião e Silva and later work.

Mario Carlos Rocca is an Argentine theoretical and mathematical physicist at the Universidad Nacional de La Plata and an Independent Researcher with Argentina's CONICET. His work sits at the crossroads of functional analysis and high-energy/gravitation, especially the use of ultradistributions of Sebastiao e Silva tambien conocidas como ultrahyper functions (refinements and extensiones generalizadas of Schwartz distributions) in quantum field theory. He co-developed influential formalisms on the convolution of ultradistributions with the late C. G. Bollini and has continued that line with collaborators such as Angel Plastino. Recent papers push ultrahyperfunction-based methods into Einstein gravity, black holes, and dark-matter-adjacent questions, including co-authorships with Mir Hameeda and Behnam Pourhassan.

Scott Douglas Jacobsen: In your 1998 paper with Bollini and Escobar, how do you define the convolution of two tempered ultradistributions?

Mario Carlos Rocca: Explaining what a tempered ultradistribution or an exponential ultradistribution is, [1] is practically impossible with words alone. To solve the problem, I have added some sections taken from my papers in which I explain what tempered ultradistributions and exponential ultradistributions are.

The following definitions have been taken almost verbatim from reference [2]. The space H of test functions such that $e^{p|x|}|D^q\phi(x)|$ is bounded for any p and q is defined (ref.[2]) by means of the countably set of norms:

$$\|\hat{\phi}\|_p'' = \sup_{0 \leq q \leq p, x} e^{p|x|} |D^q \hat{\phi}(x)| \quad , \quad p = 0, 1, 2, \dots \quad (2.1)$$

According to reference[3] H is a $\mathcal{K}\{M_p\}$ space with:

$$M_p(x) = e^{(p-1)|x|} \quad , \quad p = 1, 2, \dots \quad (2.2)$$

$\mathcal{K}\{e^{(p-1)|x|}\}$ satisfies condition ($\mathcal{N}$) of Guelfand (ref.[4]). It is a countable Hilbert and nuclear space:

$$\mathcal{K}\{e^{(p-1)|x|}\} = H = \bigcap_{p=1}^{\infty} H_p \quad (2.3)$$

where H_p is obtained by completing H with the norm induced by the scalar product:

$$\langle \hat{\phi}, \hat{\psi} \rangle_p = \int_{-\infty}^{\infty} e^{2(p-1)|x|} \sum_{q=0}^p D^q \bar{\hat{\phi}}(x) D^q \hat{\psi}(x) dx \quad ; \quad p = 1, 2, \dots \quad (2.4)$$

If we take the usual scalar product:

$$\langle \hat{\phi}, \hat{\psi} \rangle = \int_{-\infty}^{\infty} \bar{\hat{\phi}}(x) \hat{\psi}(x) dx \quad (2.5)$$

then H , completed with (2.5), is the Hilbert space $\mathcal{H}$ of square integrable functions.

The space of continuous linear functionals defined on H is the space Λ_{∞} of the distributions of the exponential type (ref.[2]).

The “nested space”

$$(H, \mathcal{H}, \Lambda_{\infty}) \quad (2.6)$$

is a Guelfand’s triplet (or a Rigged Hilbert space [4]).

Any Guelfand’s triplet $(\mathcal{A}, \mathcal{H}, \mathcal{A}')$ has the fundamental property that a linear and symmetric operator on $\mathcal{A}$, admitting an extension to a self-adjoint operator in $\mathcal{H}$, has a complete set of generalized eigenfunctions in $\mathcal{A}'$ with real eigenvalues.

3 Tempered Ultradistributions

The Fourier transform of a function $\hat{\phi} \in H$ is

$$\phi(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dx e^{izx} \hat{\phi}(x) \quad (3.1)$$

$\phi(z)$ is entire analytic and rapidly decreasing on straight lines parallel to the real axis. We shall call h the set of all such functions.

$$h = \mathcal{F}\{H\} \quad (3.2)$$

It is a $\mathcal{L}\{M_p\}$ space (ref.[3]), countably normed and complete, with:

$$M_p(z) = (1 + |z|)^p \quad (3.3)$$

h is also a nuclear space with norms:

$$\|\phi\|_{pn} = \sup_{|Im(z)| \leq n} (1 + |z|)^p |\phi(z)| \quad (3.4)$$

We can define the usual scalar product:

$$\langle \phi(z), \psi(z) \rangle = \int_{-\infty}^{\infty} \phi(z) \psi_1(z) dz = \int_{-\infty}^{\infty} \hat{\phi}(x) \hat{\psi}(x) dx \quad (3.5)$$

where:

$$\psi_1(z) = \int_{-\infty}^{\infty} dx e^{-izx} \hat{\psi}(x)$$

By completing h with the norm induced by (3.5) we get the Hilbert space of square integrable functions.

The dual of h is the space $\mathcal{U}$ of tempered ultradistributions (ref.[2]). In other words, a tempered ultradistribution is a continuous linear functional defined on the space h of entire functions rapidly decreasing on stright lines parallel to the real axis.

The set $(h, \mathcal{H}, \mathcal{U})$ is also a Guelfand's triplet.

$\mathcal{U}$ can also be characterized in the following way (ref.[2]): let $\mathcal{A}$ be the space of all functions $F(z)$ such that:

I- $F(z)$ is analytic for $\{z \in \mathcal{C} : |Im(z)| > p\}$.

II- $F(z)/z^p$ is bounded continuous in $\{z \in \mathcal{C} : |Im(z)| \geq p\}$, where $p = 0, 1, 2, \dots$ depends on $F(z)$.

Let Π be the set of all z -dependent polynomials, $z \in \mathcal{C}$. Then $\mathcal{U}$ is the quotient space:

III- $\mathcal{U} = \mathcal{A} / \Pi$

Due to these properties it is possible to represent any ultradistribution as (ref.[2]):

$$F(\phi) = \langle F(z), \phi(z) \rangle = \oint_{\Gamma} dz F(z) \phi(z) \quad (3.6)$$

where the path Γ runs parallel to the real axis from $-\infty$ to ∞ for $Im(z) > \rho$, $\rho > p$ and back from ∞ to $-\infty$ for $Im(z) < -\rho$, $-\rho < -p$. (Γ lies outside a horizontal band of width $2p$ containing all the singularities of $F(z)$).

Formula (3.6) will be our fundamental representation for a tempered ultradistribution. Sometimes use will be made of "Dirac formula" for ultradistributions (ref.[1]):

$$F(z) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} dt \frac{f(t)}{t - z} \quad (3.7)$$

where the “density” $f(t)$ is such that

$$\oint_{\Gamma} dz F(z) \phi(z) = \int_{-\infty}^{\infty} dt f(t) \phi(t) \quad (3.8)$$

While $F(z)$ is analytic on Γ , the density $f(t)$ is in general singular, so that the r.h.s. of (3.8) should be interpreted in the sense of distribution theory.

Another important property of the analytic representation is the fact that on Γ , $F(z)$ is bounded by a power of z (ref.[2]):

$$|F(z)| \leq C|z|^p \quad (3.9)$$

where C and p depend on F .

The representation (3.6) makes evident that the addition of a polynomial $P(z)$ to $F(z)$ do not alter the ultradistribution:

$$\oint_{\Gamma} dz \{F(z) + P(z)\} \phi(z) = \oint_{\Gamma} dz F(z) \phi(z) + \oint_{\Gamma} dz P(z) \phi(z)$$

But:

$$\oint_{\Gamma} dz P(z) \phi(z) = 0$$

as $P(z)\phi(z)$ is entire analytic (and rapidly decreasing),

$$\therefore \oint_{\Gamma} dz \{F(z) + P(z)\} \phi(z) = \oint_{\Gamma} dz F(z) \phi(z) \quad (3.10)$$

4 The Convolution

If we try to define the convolution product by means of the natural formula:

$$(F * G)\{\phi\} = \oint_{\Gamma_1} \oint_{\Gamma_2} dk_1 dk_2 F(k_1) G(k_2) \phi(k_1 + k_2) \quad (4.1)$$

we will soon discover that it is not always defined. The reason is simple. The result of:

$$\oint_{\Gamma} dk F(k) \phi(k + k') = \chi(k')$$

does not, in general, belong to h . However, if at least one of the ultradistributions F and G is rapidly decreasing (say G), then a convolution can be defined (ref.[1]) by:

$$H(k) = \int_{-\infty}^{\infty} dt f(t) G(k - t) \quad (4.2)$$

where $f(t)$ is the density associated to $F(k)$ (cf.(3.7)).

In order to eliminate the test function from (4.1) use can be made of the complex δ -function, which is an ultradistribution (Cauchy's theorem):

$$\delta_{z'}\{\phi\} = -\frac{1}{2\pi i} \oint_{\Gamma} dz \frac{\phi(z)}{z - z'} = \phi(z') \quad (4.3)$$

where the point z' is enclosed by Γ . (This procedure was previously used in ref.[5]). We can then write (4.1) as:

$$(F * G)\{\phi\} = -\frac{1}{2\pi i} \oint_{\Gamma} dz \oint_{\Gamma_1} \oint_{\Gamma_2} dk_1 dk_2 \frac{F(k_1) G(k_2)}{z - k_1 - k_2} \phi(z) \quad (4.4)$$

The path Γ must have:

$$|Im(z)| > |Im(k_1)| + |Im(k_2)| \quad (4.5)$$

in order to embrace the point $k_1 + k_2$, ($k_1 \in \Gamma_1$, $k_2 \in \Gamma_2$).

Equation (4.4) leads to:

$$F * G = H = -\frac{1}{2\pi i} \oint_{\Gamma_1} \oint_{\Gamma_2} dk_1 dk_2 \frac{F(k_1)G(k_2)}{z - k_1 - k_2} \quad (4.6)$$

However, we do not expect (4.6) to define a tempered ultradistribution for every pair F, G . Note that in (4.1) F and G operate on $\phi(k)$ which is rapidly decreasing, while in (4.6) they act on $(z - k)^{-1}$ ($k = k_1 + k_2$). Furthermore, due to (4.5) and the fact that Γ_1 and Γ_2 run outside a horizontal band containing all the singularities of F and G , the integrand in (4.6) is analytic at every point of the integration paths. Taking into account the property (3.9) of tempered ultradistributions, we come to the conclusion that the integrations in (4.6) have at most, a tempered singularity for $k \rightarrow \infty$. In order to control this possible singularity we introduce a regulator (see ref.[6]).

We define:

$$H_\lambda(z) = \frac{i}{2\pi} \oint_{\Gamma_1} \oint_{\Gamma_2} dk_1 dk_2 \frac{k_1^\lambda F(k_1) k_2^\lambda G(k_2)}{z - k_1 - k_2} \quad (4.7)$$

Now, if we have the bounds:

$$|F(k_1)| \leq C_1 |k_1|^m ; \quad |G(k_2)| \leq C_2 |k_2|^n \quad (4.8)$$

Then (4.7) is convergent for

$$Re(\lambda) < -l - 1 ; \quad l = \max\{m, n\} \quad (4.9)$$

It is also analytic in the region (4.9) of the λ plane. As the derivative with respect to λ merely multiplies by a logarithmic factor the integrand of (4.7) without spoiling the convergence.

According to the method of ref.[6], H_λ can be analytically continued to other parts of the λ plane. In particular near the origin we have the Laurent (or Taylor) expansion:

$$H_\lambda = \sum_n H^{(n)}(z) \lambda^n \quad (4.10)$$

where the sum might have terms with negative n . We now define the convolution product as the λ -independent term of (4.10):

$$H(z) = H^{(0)}(z) \quad (4.11)$$

Note that the derivatives of $H_\lambda(z)$ with respect to z can be obtained from (4.7) by taking different powers of the denominator:

$$\frac{d^p H_\lambda(z)}{dz^p} = (-1)^p p! \frac{i}{2\pi} \oint_{\Gamma_1} \oint_{\Gamma_2} dk_1 dk_2 \frac{k_1^\lambda F(k_1) k_2^\lambda G(k_2)}{(z - k_1 - k_2)^p} \quad (4.12)$$

The convergence of (4.7) also ensures that of (4.12). Therefore, also ensure analyticity in z , outside the horizontal band defined by (4.5). We will now show that $|H_\lambda(z)|$ is bounded by a power of $|z|$ (cf.(3.9)).

To that aim we take:

$$\begin{aligned} Im(\lambda) &= 0 ; \quad \lambda < -l - 1 ; \quad z = x + iy \\ k_i &= \kappa_i \pm i\sigma_i ; \quad \sigma_i > 0 ; \quad dk_i = d\kappa_i \end{aligned}$$

The integrals along Γ_i , can be expressed as integrals on $d\kappa_i$ between $0 \rightarrow \infty$. Then we have:

$$\begin{aligned} |H_\lambda| &= \frac{1}{2\pi} \left| \oint_{\Gamma_1} \oint_{\Gamma_2} dk_1 dk_2 \frac{k_1^\lambda F(k_1) k_2^\lambda G(k_2)}{z - k_1 - k_2} \right| \leq \\ &\frac{1}{2\pi} \oint_{\Gamma_1} \oint_{\Gamma_2} \text{sgn} Im(k_1) dk_1 \text{sgn} Im(k_2) dk_2 \frac{|k_1|^\lambda C_1 |k_1|^m |k_2|^\lambda C_2 |k_2|^n}{|z - k_1 - k_2|} \leq \end{aligned}$$

$$\begin{aligned} \frac{C_1 C_2}{2\pi} \oint_{\Gamma_1} \oint_{\Gamma_2} \text{sgnIm}(k_1) dk_1 \text{sgnIm}(k_2) dk_2 |k_1|^{\lambda+m} |k_2|^{\lambda+n} = \\ \frac{8C_1 C_2}{\pi} \int_0^\infty \int_0^\infty d\kappa_1 d\kappa_2 (\kappa_1^2 + \sigma_1^2)^{\frac{\lambda+m}{2}} (\kappa_2^2 + \sigma_2^2)^{\frac{\lambda+n}{2}} \end{aligned} \quad (4.13)$$

We make now the change of variables $w_i = \kappa_i^2$ and obtain:

$$(4.13) = \frac{2C_1 C_2}{\pi} \int_0^\infty dw_1 w_1^{-\frac{1}{2}} (w_1 + \sigma_1^2)^{\frac{\lambda+m}{2}} \int_0^\infty dw_2 w_2^{-\frac{1}{2}} (w_2 + \sigma_2^2)^{\frac{\lambda+n}{2}} = \quad (4.14)$$

$$\begin{aligned} \frac{2C_1 C_2}{\pi} \mathcal{B}\left(\frac{1}{2}, -\frac{\lambda+m+1}{2}\right) \mathcal{B}\left(\frac{1}{2}, -\frac{\lambda+n+1}{2}\right) \sigma_1^{\frac{\lambda+m+1}{2}} \sigma_2^{\frac{\lambda+n+1}{2}} \leq \\ C(\lambda, m, n) |z|^{\lambda+m+n+1} \end{aligned} \quad (4.15)$$

where $\mathcal{B}(x, y)$ is Gauss beta function.

It is to be noted that if $G(k)$ is a rapidly decreasing ultradistribution, then $H_\lambda(z)$ (eq.(4.7)) coincides with $H_0(z)$:

$$H_0(z) = \frac{i}{2\pi} \oint_{\Gamma_1} dk_1 F(k_1) \oint_{\Gamma_2} dk_2 \frac{G(k_2)}{z - k_1 - k_2} \quad (4.16)$$

In fact, near $\lambda = 0$ we have ($|k| > 1$):

$$|k^\lambda - 1| \leq \lambda(2\pi + |\ln|k||) |k|^\lambda \quad (4.17)$$

$$\begin{aligned} H_\lambda - H_0(z) = \frac{i}{2\pi} \oint_{\Gamma_1} dk_1 k_1^\lambda F(k_1) \oint_{\Gamma_2} dk_2 (k_2^\lambda - 1) \frac{G(k_2)}{z - k_1 - k_2} + \\ \frac{i}{2\pi} \oint_{\Gamma_1} dk_1 (k_1^\lambda - 1) F(k_1) \oint_{\Gamma_2} dk_2 \frac{G(k_2)}{z - k_1 - k_2} \end{aligned} \quad (4.18)$$

In eq.(4.18) the integrals are convergent, as $G(k)$ and $k^\lambda G(k)$ are both rapidly decreasing. Furthermore, due to (4.17) the difference $H_\lambda - H_0$ is proportional to λ . So that

$$\lim_{\lambda \rightarrow 0} [H_\lambda - H_0] = 0 \quad (4.19)$$

Again, when $G(k)$ is rapidly decreasing, the convolution defined in ref.[8]:

$$H(z) = \int_{-\infty}^{\infty} dt f(t) G(z-t) \quad (4.20)$$

(where $f(t)$ is given by (3.7),(3.8)), also coincides with (4.16). To show that (4.16) implies (4.20), we use (3.8) in (4.16)

$$H_0(z) = \frac{i}{2\pi} \int_{-\infty}^{\infty} dt f(t) \oint_{\Gamma_2} dk_2 \frac{G(k_2)}{z - k_1 - k_2}$$

But, if $G(t)$ is the density associated to $G(z)$, then

$$\frac{i}{2\pi} \oint_{\Gamma_2} dk_2 \frac{G(k_2)}{z - t - k_2} = \frac{1}{2\pi i} \int_{-\infty}^{\infty} dt_2 \frac{g(t_2)}{t_2 - (z-t)} = G(z-t)$$

I.e.:

$$H_0(z) = H(z) \quad (4.21)$$

5 Ultradistributions of Exponential Type

Let $\mathcal{S}$ be the space of Schwartz of test functions rapidly decreasing. Let Λ_j be the region of the complex plane defined as:

$$\Lambda_j = \{z \in \mathbb{C} : |\Im(z)| < j : j \in \mathbb{N}\} \quad (5.22)$$

According to ref.[7, 1] the space of test functions $\hat{\phi} \in \mathcal{V}_j$ is constituted by all entire analytic functions of $\mathcal{S}$ for which

$$\|\hat{\phi}\|_j = \max_{k \leq j} \left\{ \sup_{z \in \Lambda_j} \left[e^{(j|\Re(z)|)} |\hat{\phi}^{(k)}(z)| \right] \right\} \quad (5.23)$$

is finite.

The space $\mathcal{Z}$ is then defined as:

$$\mathcal{Z} = \bigcap_{j=0}^{\infty} \mathcal{V}_j \quad (5.24)$$

It is a complete countably normed space with the topology generated by the system of semi-norms $\{\|\cdot\|_j\}_{j \in \mathbb{N}}$. The dual of $\mathcal{Z}$, denoted by $\mathfrak{B}$, is by definition the space of ultradistributions of exponential type (ref.[7, 1]). Let $\mathfrak{S}$ the space of rapidly decreasing sequences. According to ref.[2] $\mathfrak{S}$ is a nuclear space. We consider now the space of sequences $\mathfrak{P}$ generated by the Taylor development of $\hat{\phi} \in \mathcal{Z}$

$$\mathfrak{P} = \left\{ \mathfrak{Q} : \mathfrak{Q} \left(\hat{\phi}(0), \hat{\phi}'(0), \frac{\hat{\phi}''(0)}{2}, \dots, \frac{\hat{\phi}^{(n)}(0)}{n!}, \dots \right) : \hat{\phi} \in \mathcal{Z} \right\} \quad (5.25)$$

The norms that define the topology of $\mathfrak{P}$ are given by:

$$\|\hat{\phi}\|_p' = \sup_n \frac{n^p}{n} |\hat{\phi}^{(n)}(0)| \quad (5.26)$$

$\mathfrak{P}$ is a subspace of $\mathfrak{S}$ and therefore is a nuclear space. As the norms $\|\cdot\|_j$ and $\|\cdot\|_p'$ are equivalent, the correspondence

$$\mathcal{Z} \longleftrightarrow \mathfrak{P} \quad (5.27)$$

is an isomorphism and therefore $\mathcal{Z}$ is a countably normed nuclear space. We can define now the set of scalar products

$$\begin{aligned} \langle \hat{\phi}(z), \hat{\psi}(z) \rangle_n &= \sum_{q=0}^n \int_{-\infty}^{\infty} e^{2n|z|} \overline{\hat{\phi}^{(q)}(z)} \hat{\psi}^{(q)}(z) dz = \\ &= \sum_{q=0}^n \int_{-\infty}^{\infty} e^{2n|x|} \overline{\hat{\phi}^{(q)}(x)} \hat{\psi}^{(q)}(x) dx \end{aligned} \quad (5.28)$$

This scalar product induces the norm

$$\|\hat{\phi}\|_n'' = [\langle \hat{\phi}(x), \hat{\phi}(x) \rangle_n]^{1/2} \quad (5.29)$$

The norms $\|\cdot\|_j$ and $\|\cdot\|_n''$ are equivalent, and therefore $\mathcal{Z}$ is a countably hilbertian nuclear space. Thus, if we call now $\mathcal{Z}_p$ the completion of $\mathcal{Z}$ by the norm p given in (5.29), we have:

$$\mathcal{Z} = \bigcap_{p=0}^{\infty} \mathcal{Z}_p \quad (5.30)$$

where

$$\mathcal{Z}_0 = H \quad (5.31)$$

is the Hilbert space of square integrable functions.

As a consequence the “nested space”

$$\mathfrak{A} = (\mathcal{Z}, H, \mathfrak{B}) \quad (5.32)$$

is a Gelfand’s triplet (or a Rigged Hilbert space=RHS. See ref.[4]).

$\mathfrak{B}$ can also be characterized in the following way (refs.[7],[8]): let $\mathfrak{E}_\omega$ be the space of all functions $\hat{F}(z)$ such that:

I- $\hat{F}(z)$ is analytic for $\{z \in \mathbb{C} : |\operatorname{Im}(z)| > p\}$.

III- $\hat{F}(z)e^{-p|\operatorname{Re}(z)|}/z^p$ is bounded continuous in $\{z \in \mathbb{C} : |\operatorname{Im}(z)| \geq p\}$, where $p = 0, 1, 2, \dots$ depends on $\hat{F}(z)$.

Let $\mathfrak{H}$ be: $\mathfrak{H} = \{\hat{F}(z) \in \mathfrak{E}_\omega : \hat{F}(z) \text{ is entire analytic}\}$. Then $\mathfrak{B}$ is the quotient space:

III- $\mathfrak{B} = \mathfrak{E}_\omega / \mathfrak{H}$

Due to these properties it is possible to represent any ultradistribution as (ref.[7, 8]):

$$\hat{F}(\hat{\phi}) = \langle \hat{F}(z), \hat{\phi}(z) \rangle = \oint_{\Gamma} \hat{F}(z) \hat{\phi}(z) dz \quad (5.33)$$

where the path Γ_j runs parallel to the real axis from $-\infty$ to ∞ for $\operatorname{Im}(z) > \zeta$, $\zeta > p$ and back from ∞ to $-\infty$ for $\operatorname{Im}(z) < -\zeta$, $-\zeta < -p$. (Γ surrounds all the singularities of $\hat{F}(z)$).

Formula (5.33) will be our fundamental representation for a tempered ultradistribution. Sometimes use will be made of "Dirac formula" for exponential ultradistributions (ref.[1]):

$$\hat{F}(z) \equiv \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{\hat{f}(t)}{t-z} dt \equiv \frac{\cosh(\lambda z)}{2\pi i} \int_{-\infty}^{\infty} \frac{\hat{f}(t)}{(t-z) \cosh(\lambda t)} dt \quad (5.34)$$

where the "density" $\hat{f}(t)$ is such that

$$\oint_{\Gamma} \hat{F}(z) \hat{\phi}(z) dz = \int_{-\infty}^{\infty} \hat{f}(t) \hat{\phi}(t) dt \quad (5.35)$$

(5.34) should be used carefully in this case. While $\hat{F}(z)$ is analytic on Γ , the density $\hat{f}(t)$ is in general singular, so that the r.h.s. of (5.35) should be interpreted in the sense of distribution theory.

Another important property of the analytic representation is the fact that on Γ , $\hat{F}(z)$ is bounded by an exponential and a power of z (ref.[7, 8]):

$$|\hat{F}(z)| \leq C|z|^p e^{p|\operatorname{Re}(z)|} \quad (5.36)$$

where C and p depend on $\hat{F}$.

The representation (5.33) implies that the addition of any entire function $\hat{G}(z \in \mathfrak{H})$ to $\hat{F}(z)$ does not alter the ultradistribution:

$$\oint_{\Gamma} \{\hat{F}(z) + \hat{G}(z)\} \hat{\phi}(z) dz = \oint_{\Gamma} \hat{F}(z) \hat{\phi}(z) dz + \oint_{\Gamma} \hat{G}(z) \hat{\phi}(z) dz$$

But:

$$\oint_{\Gamma} \hat{G}(z) \hat{\phi}(z) dz = 0$$

as $\hat{G}(z)\hat{\phi}(z)$ is entire analytic (and rapidly decreasing),

$$\therefore \oint_{\Gamma} \{\hat{F}(z) + \hat{G}(z)\} \hat{\phi}(z) dz = \oint_{\Gamma} \hat{F}(z) \hat{\phi}(z) dz \quad (5.37)$$

Another very important property of $\mathfrak{B}$ is that $\mathfrak{B}$ is reflexive under the Fourier transform:

$$\mathfrak{B} = \mathcal{F}_c \{ \mathfrak{B} \} = \mathcal{F} \{ \mathfrak{B} \} \quad (5.38)$$

where the complex Fourier transform $F(k)$ of $\hat{F}(z) \in \mathfrak{B}$ is given by:

$$\begin{aligned} F(k) &= \Theta[\Im(k)] \int_{\Gamma_+} \hat{F}(z) e^{ikz} dz - \Theta[-\Im(k)] \int_{\Gamma_-} \hat{F}(z) e^{ikz} dz = \\ &= \Theta[\Im(k)] \int_0^{\infty} \hat{f}(x) e^{ikx} dx - \Theta[-\Im(k)] \int_{-\infty}^0 \hat{f}(x) e^{ikx} dx \end{aligned} \quad (5.39)$$

Here Γ_+ is the part of Γ with $\Re(z) \geq 0$ and Γ_- is the part of Γ with $\Re(z) \leq 0$ Using (5.39) we can interpret Dirac's formula as:

$$F(k) \equiv \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{f(s)}{s-k} ds \equiv \mathcal{F}_c \{ \mathcal{F}^{-1} \{ f(s) \} \} \quad (5.40)$$

The treatment for ultradistributions of exponential type defined on $\mathbb{C}^n$ is similar to the one-variable. In this case

$$\Lambda_j = \{z = (z_1, z_2, \dots, z_n) \in \mathbb{C}^n : |\Im(z_k)| \leq j \quad 1 \leq k \leq n\} \quad (5.41)$$

$$\|\hat{\phi}\|_j = \max_{k \leq j} \left\{ \sup_{z \in \Lambda_j} \left[e^{j \left[\sum_{p=1}^n |\Re(z_p)| \right]} \left| D^{(k)} \hat{\phi}(z) \right| \right] \right\} \quad (5.42)$$

where $D^{(k)} = \partial^{(k_1)} \partial^{(k_2)} \dots \partial^{(k_n)} \quad k = k_1 + k_2 + \dots + k_n$

$\mathfrak{H}$ is characterized as follows. Let $\mathfrak{E}_\omega$ be the space of all functions $\hat{F}(z)$ such that:

I' - $\hat{F}(z)$ is analytic for $\{z \in \mathbb{C}^n : |\Im(z_1)| > p, |\Im(z_2)| > p, \dots, |\Im(z_n)| > p\}$.

II' - $\hat{F}(z) e^{-\left[p \sum_{j=1}^n |\Re(z_j)| \right]} / z^p$ is bounded continuous in $\{z \in \mathbb{C}^n : |\Im(z_1)| \geq p, |\Im(z_2)| \geq p, \dots, |\Im(z_n)| \geq p\}$, where $p = 0, 1, 2, \dots$ depends on $\hat{F}(z)$.

Let $\mathfrak{H}$ be: $\mathfrak{H} = \{ \hat{F}(z) \in \mathfrak{E}_\omega : \hat{F}(z) \text{ is entire analytic at minus in one of the variables } z_j \quad 1 \leq j \leq n \}$ Then $\mathfrak{H}$ is the quotient space:

$III' - \mathfrak{H} = \mathfrak{E}_\omega / \mathfrak{H}$ We have now

$$\hat{F}(\hat{\phi}) = \langle \hat{F}(z), \hat{\phi}(z) \rangle = \oint_{\Gamma} \hat{F}(z) \hat{\phi}(z) dz_1 dz_2 \dots dz_n \quad (5.43)$$

$\Gamma = \Gamma_1 \cup \Gamma_2 \cup \dots \cup \Gamma_n$ where the path Γ_j runs parallel to the real axis from $-\infty$ to ∞ for $\Im(z_j) > \zeta$, $\zeta > p$ and back from ∞ to $-\infty$ for $\Im(z_j) < -\zeta$, $-\zeta < -p$. (Again Γ surrounds all the singularities of $\hat{F}(z)$). The n-dimensional Dirac's formula is

$$\hat{F}(z) = \frac{1}{(2\pi i)^n} \int_{-\infty}^{\infty} \frac{\hat{f}(t)}{(t_1 - z_1)(t_2 - z_2) \dots (t_n - z_n)} dt_1 dt_2 \dots dt_n \quad (5.44)$$

where the "density" $\hat{f}(t)$ is such that

$$\oint_{\Gamma} \hat{F}(z) \hat{\phi}(z) dz_1 dz_2 \dots dz_n = \int_{-\infty}^{\infty} f(t) \hat{\phi}(t) dt_1 dt_2 \dots dt_n \quad (5.45)$$

and the modulus of $\hat{F}(z)$ is bounded by

$$|\hat{F}(z)| \leq C |z|^p e^{\left[p \sum_{j=1}^n |\Re(z_j)| \right]} \quad (5.46)$$

where C and p depend on $\hat{F}$.

6 The Convolution of two Ultradistributions of Exponential Type

The convolution of two ultradistributions of exponential type can be defined with a change in the formula obtained for tempered ultradistributions Let here be

$$H_{\gamma\lambda}(k) = \frac{i}{2\pi} \oint_{\Gamma_1} \oint_{\Gamma_2} \frac{[2 \cosh(\gamma k_1)]^{-\lambda} F(k_1) [2 \cosh(\gamma k_2)]^{-\lambda} G(k_2)}{k - k_1 - k_2} dk_1 dk_2 \quad (6.1)$$

$$|\Im(k)| > |\Im(k_1)| + |\Im(k_2)|; \gamma < \min \left(\frac{\pi}{2 |\Im(k_1)|}; \frac{\pi}{2 |\Im(k_2)|} \right)$$

With this value of γ the hyperbolic cosines has not singularities in the integration zone. Again we have the Laurent (or Taylor) expansion:

$$H_{\gamma\lambda}(k) = \sum_n H_{\gamma}^{(n)}(k) \lambda^n \quad (6.2)$$

where the sum might have terms with negative n. We now define the convolution product as the λ -independent term of (6.2)

$$(F * G)(k) = H(k) = H_{\gamma}^{(0)}(k) = H^{(0)}(k) \quad (6.3)$$

that is γ -independent

To see this we consider a typical integral term in (6.1)

$$I = \int_c^{\infty} \frac{F(k+i\sigma)}{[\cosh \gamma(k+i\sigma)]^{\lambda}} dk \quad (6.4)$$

with

$$|F(k)| \leq A |k|^p e^{p|\Re(k)|} \quad (6.5)$$

Then I has the value

$$I = e^{i(p+1)} \left[\sum_{n=1; n \neq (p-1, p+1)}^{\infty} a_n(p, \sigma) \frac{e^{-c(n-p-1)}}{n-p-1} - a_{p-1}(p, \sigma) \frac{e^{2c}}{2} + \frac{a_{p+1}(p, \sigma)}{\lambda \gamma} \right] \quad (6.6)$$

Thus the λ -independent term of I does not depend of γ . As (6.1) is composed by sums and products of integrals of the type (6.4) we conclude that (6.3) is true.

Jacobsen: In that 1998 framework, how do you construct the product of two distributions of exponential type?

Rocca: The product of two exponential distributions is the Fourier anti-transform of the two tempered ultradistributions, that is, a product on a ring with zero divisors.

Jacobsen: Following from the previous question, what minimal conditions ensure existence and associativity?

Rocca: The existence of the product is established constructively, with the corresponding theorems (see the beginning of this note). The product is not generally associative, since it is a product in a ring with zero divisors. The product must be performed carefully, taking into account the physical conditions of the problem.

Jacobsen: Using that four-dimensional result, what is the explicit finite expression for the convolution of two Wheeler propagators in massless and complex-mass cases?

Rocca:

For a massless particle we have:

$$[w_0 * w_0](k^0, \rho) = \frac{\pi^3}{2} \quad (6.7)$$

where k^0 and ρ are real variables.

And for a complex-mass particle we have:

$$[W_{\mu_1} * W_{\mu_2}](k^0, \rho) = \frac{\pi^3 \text{Sgn}[\Im(k^0)]}{4(k_0^2 - \rho^2)} \sqrt{(k_0^2 - \rho^2 + \mu_2^2 - \mu_1^2)^2 - 4(k_0^2 - \rho^2)\mu_2^2} \quad (6.8)$$

where k^0 is a complex variable and ρ is a real variable.

Jacobsen: What necessary and sufficient analyticity and support conditions in the ultrahyperfunction setting guarantee microcausality for field commutators?

Rocca: This is guaranteed, since quantum fields are vector ultradistributions. Their product is defined from the usual ultradistributions. A vector ultradistribution is a continuous functional defined on a space of test functions and taking values in a locally convex topological vector space. Operator valued distributions are a special case of vector ultradistributions.

Jacobsen: Which growth and analyticity conditions on propagators treated as ultrahyperfunctions ensure finiteness of loop integrals without renormalization?

Rocca: All propagators known so far are ultradistributions. If they are exponentially increasing propagators, they are exponential ultradistributions. This ensures the finiteness of the integral loops.

Jacobsen: What is Einstein Gravity?

Rocca: It is the geometry of space-time created by the presence of masses.

Jacobsen: What is Gupta–Feynman–based on the QFT of Einstein gravity?

Rocca: This is the gravity proposed by Gupta and Feynman by developing the graviton field into powers of the gravitational constant and quantizing it using the Gupta-Bleuler method. The best-known case is the linear approximation, which is the case I solved exactly with Mir Hameeda and Angelo Plastino.

Jacobsen: Following from the last two questions, which constraint and gauge conditions maintain S-matrix unitarity while avoiding Faddeev-Popov ghosts?

Rocca: In the case we are dealing with, the quantization was made unitary by adding a simple constraint. So far, I have had the experience that if the most general quantization method, the Feynman-Schwinger Variational Principle, is used, the ghosts do not appear in the theory being treated.

Jacobsen: How does the ultrahyperfunction-based quantization program relate to effective field theory?

Rocca: The theory of ultrahyperfunctions is used to quantize fundamental theories. However, using the same method, one can also quantize effective theories. For me, Einstein's theory of gravity is a fundamental theory. To quantize this theory, a very rigorous mathematical theory must be used, like the one we use.

Jacobsen: What are infrared and massless regimes?

Rocca: The infrared regime corresponds to small momentums. The massless regime is obtained for massive particles outside the mass-shell.

Jacobsen: For these, how are soft and collinear singularities handled within the ultradistribution and ultrahyperfunction formalism?

Rocca: In the theory of ultrahyperfunctions, singularities do not exist.

Jacobsen: What is Minkowski space?

Rocca: It is simply a semi-Riemannian manifold with a particular metric.

Jacobsen: Following from the previous questions, which parts of the ultrahyperfunction construction extend from Minkowski space to general globally hyperbolic curved spacetimes?

Rocca: The construction of ultradistributions for any semi-Riemannian manifold is analogous to the construction in Minkowski space, only with another metric and other variables.

Jacobsen: In your BTZ-background studies, how is the graviton self-energy computed in 2+1 and 3+1 dimensions?

Here are the expressions. In 3+1 dimensions it is:

$$\begin{aligned}
& k_{\alpha_1} k_{\alpha_2} (\rho - m^2 + i0)^{-1} * k_{\alpha_3} k_{\alpha_4} (\rho - m^2 + i0)^{-1} = \\
& \frac{i\pi^2}{64} [\eta_{\alpha_1 \alpha_2} \eta_{\alpha_3 \alpha_4} + \eta_{\alpha_1 \alpha_3} \eta_{\alpha_2 \alpha_4} + \eta_{\alpha_1 \alpha_4} \eta_{\alpha_2 \alpha_3}] \left(\frac{5}{2} \rho^2 - 4m^2 \rho + \frac{9}{4} m^4 \right) \\
& + \frac{i\pi^2}{8} [\eta_{\alpha_1 \alpha_2} k_{\alpha_3} k_{\alpha_4} + \eta_{\alpha_3 \alpha_4} k_{\alpha_1} k_{\alpha_2}] \left(\frac{41}{400} \rho + \frac{3}{2} m^2 \right) \\
& + \frac{i\pi^2}{8} [\eta_{\alpha_1 \alpha_3} k_{\alpha_2} k_{\alpha_4} + \eta_{\alpha_1 \alpha_4} k_{\alpha_2} k_{\alpha_3} + \eta_{\alpha_2 \alpha_3} k_{\alpha_1} k_{\alpha_4} + \eta_{\alpha_2 \alpha_4} k_{\alpha_1} k_{\alpha_3}] \left(\frac{103}{900} \rho - \frac{35}{144} m^2 \right)
\end{aligned} \tag{6.9}$$

To understand this formula, you should read the paper [9].

In 2+1 dimensions it is:

$$\begin{aligned}
& [k_{\beta_1} k_{\beta_2} (k^2 + m^2 - i0)^{-1} * k_{\beta_3} k_{\beta_4} (k^2 + m^2 - i0)^{-1}]_{(2+1)} = \\
& \left\{ \frac{i\pi^2}{64} [\theta_{\beta_1 \beta_2} \theta_{\beta_3 \beta_4} + \theta_{\beta_1 \beta_3} \theta_{\beta_2 \beta_4} + \theta_{\beta_1 \beta_4} \theta_{\beta_2 \beta_3}] \left(\frac{5}{2} - k^4 m^2 k^2 + \frac{9}{4} m^4 \right) + \right. \\
& \left. \frac{i\pi^2}{8} [\theta_{\beta_1 \beta_2} k_{\beta_3} k_{\beta_4} + \theta_{\beta_3 \beta_4} k_{\beta_1} k_{\beta_2}] \left(\frac{41}{400} k^2 + \frac{3}{2} m^2 \right) + \right. \\
& \left. \frac{i\pi^2}{8} [\theta_{\beta_1 \beta_3} k_{\beta_2} k_{\beta_4} + \theta_{\beta_1 \beta_4} k_{\beta_2} k_{\beta_3} + \theta_{\beta_2 \beta_3} k_{\beta_1} k_{\beta_4} + \theta_{\beta_2 \beta_4} k_{\beta_1} k_{\beta_3}] \left(\frac{103}{900} k^2 - \frac{35}{144} m^2 \right) \right\}_{(2+1)}.
\end{aligned} \tag{6.10}$$

where to understand this last formula you should read the paper [10].

Finally, I should clarify that ultrahyperfunctions are to distributions what complex functions are to real-world functions. That's how important their role is in next-generation rigorous mathematics.

Jacobsen: Thank you for the opportunity and your time, Marco.

Methods

The interview was conducted via typed questions—with explicit consent—for review, and curation. This process complied with applicable data protection laws, including the California Consumer Privacy Act (CCPA), Canada's Personal Information Protection and Electronic Documents Act (PIPEDA), and Europe's General Data Protection Regulation (GDPR), i.e., recordings if any were stored securely, retained only as needed, and deleted upon request, as well in accordance with Federal Trade Commission (FTC) and Advertising Standards Canada guidelines.

Data Availability

No datasets were generated or analyzed during the current article. All interview content remains the intellectual property of the interviewer and interviewee.

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How Many Regions Do Overlapping Cubes Make?

Anthropic's Claude Fable 5

Last updated 2026-07-18. This document is standalone: every term is defined here, and no other file is required to follow it.

1. The question

Take several identical cubes, all sharing the same centre point, each turned to a different angle, and imagine them made of glass so you can see all their faces at once. The faces slice space into many pieces. This project asks a simple question with a surprisingly rich answer:

With n cubes sharing a centre, how many bounded pieces can their faces cut space into, and which arrangement of angles achieves the most?

We insist on **exact** answers — whole numbers proven correct, not measured approximately — and we search for the maximizing arrangement.

Making it precise

- Each cube is the same: the axis-aligned cube whose corners are at $(\pm 1, \pm 1, \pm 1)$, then rotated about the common centre (the origin). So a configuration of n cubes is just a choice of n rotations.
- Each cube has 6 faces, each lying in a flat plane. With n cubes there are $6n$ planes. These planes divide all of space into cells.
- A cell is **bounded** if it is finite (enclosed on all sides) and **unbounded** if it stretches to infinity. There is exactly one unbounded outside region; we count only the bounded ones. "Total" = number of bounded pieces.
- Every bounded piece lies **inside** some number of the cubes. Its **depth** is that number, from 1 (inside just one cube) up to n (inside all of them). We track the whole profile, written by `_depth = {1: ..., 2: ..., ..., n: ...}`. The innermost piece — inside all n cubes at once — is always a single convex blob, so the depth- n count is always 1.

A worked feel for the numbers: one cube alone makes 1 bounded piece (itself). Two generic cubes make 13. Three make up to 67, and so on. For six cubes the count runs into the hundreds, and finding the true maximum is hard.

Specifying a rotation with four integers

A rotation in 3D can be encoded by four numbers called a *quaternion*, written (w, x, y, z) . The only facts needed here:

- Any four integers give a valid rotation, and — crucially — that rotation has **rational** entries (fractions), so it can be represented and computed with *exactly*, no rounding.
- Scaling all four by the same factor gives the same rotation, so we keep them as small whole numbers with no common factor.

Throughout, a configuration is written as a list of n integer quaternions, e.g. 4, 1, 1, -1 ; 3, 3, 7, 3 ; ... (one cube per group). Some special maximizing arrangements need rotations built from $\sqrt{2}$ or $\sqrt{5}$ instead of plain fractions; those are handled with exact arithmetic in the corresponding number system.

2. Why exact counting, and how it is done

The cautionary tale. The project began by counting pieces approximately: chop space into tiny voxels (3D pixels) and flood-fill to label connected pieces. This *cannot reliably rank arrangements*. One random configuration showed an apparent stable count near ~1340 across three resolutions — but about 70% of those "pieces" were unresolved slivers thinner than the voxel grid; its true exact count is 567. Both over-counting (one piece split into many) and under-counting (many pieces merged into one) happen at once, and a stable-looking approximate number is not proof of anything. Every result below therefore rests on exact arithmetic.

The exact method. Because each rotation is rational, each of the $6n$ face-planes has rational coefficients, and the entire arrangement of pieces can be constructed with exact fraction arithmetic:

1. Start with a large box and repeatedly cut it by each plane, splitting every cell the plane crosses into two convex cells. This yields all the convex fragments exactly.
2. A cube's face is a bounded *square*, not an infinite plane. So some fragment boundaries lie on the plane but *outside* the actual square face — these "phantom" walls are not real separations. Fragments touching across a phantom wall are the same piece and get merged (tracked with a union-find bookkeeping structure).
3. For each resulting piece, record its depth (how many cubes contain a test point inside it) and tally the results.

No step ever compares a floating-point number to zero; all decisions are exact comparisons of fractions (or of numbers of the form $a + b\sqrt{5}$, etc., when needed). A fast version written in C/C++ counts a six-cube configuration in a fraction of a second; a slower reference implementation in Python cross-checks it.

Validation. Before trusting any search, the counter must reproduce known exact values: six cubes fanned around a shared axis must give exactly $(2 \cdot 6 - 1)^2 = 121$; the five-cube "golden" compound (below) must give 351; and the fast and slow counters must agree on hundreds of random cases with zero disagreements.

3. Warm-up: the small cases

The richest reference arrangement is the **compound of five cubes inscribed in a regular dodecahedron** — five cubes whose corners sit on the 20 vertices of a dodecahedron. Its sub-compounds count exactly:

number of cubes	1	2	3	4	5
bounded pieces	1	13	67	177	351

These arrangements involve the golden ratio and are represented exactly using numbers of the form $a + b\sqrt{5}$. They are the best known for two and three cubes — but *not* for four or five: the golden four-cube compound (177) is beaten by a purely rational four-cube arrangement reaching **183**, and the golden five-cube compound (351) is beaten by a rational five-cube arrangement reaching **393** — which is, strikingly, a *sub-compound of the best six-cube arrangement* (drop one of its six cubes). This is the same surprise as at six cubes, where the best rational arrangement (723) beats the golden-based one (681): the golden compound is the best *symmetric* configuration, not the overall maximum. So the records for four and five cubes are at least 183 and 393, and none of these is proven maximal.

The records nest. The best six-cube arrangement (723) contains the best four-cube one (183) as a sub-compound, contains a five-cube sub-compound (393) that beats the old five-cube record, and its pairs reach the two-cube record (13). Outstanding arrangements are built from outstanding smaller ones — which suggests constructing a record for n cubes by taking the best $(n-1)$ -cube arrangement and adding one more cube well. (Curiously the nesting is cleanest at even sizes: the three-cube sub-compounds of 723 reach only 63, short of the golden 67, because that maximum needs a symmetry 723's rational structure doesn't share.)

Local perfection is globally frustrated past three cubes. The golden compound has a remarkable property: *every* one of its sub-compounds is optimal — every pair is a best pair (13), every triple a best triple (67). That is exactly what forces its high symmetry, and up to three cubes it also makes it the overall maximum. But from four cubes on, the property *backfires*: the golden four-compound has every subset optimal yet totals only 177, while the true maximum (183) has to *detune* — its triples drop to 63, half its pairs to 9 — to win. You cannot both make every part optimal and maximize the whole; it's a frustration, like a magnet whose neighbours all want to align but can't all be satisfied at once. The reason it switches on at four cubes is the appearance of a *middle* depth layer: golden loads the outermost layer (depth-1) heavily — more than the true maxima do — and maxes the deep layers that are capped at $6n$, but four cubes is the first size with a depth layer that is neither — and there the maximum beats golden by shifting count out of the outer layer into the middle. This one idea ties together why golden loses, why the records keep only a modest 3-fold symmetry instead of the full icosahedral one, and why building from a record (which already carries this detuning) beats assembling from

perfect parts. At the level of the parts themselves the accounting is exact: the record's three-cube pieces count $63 = \{44, 18, 1\}$ against the perfect triple's $67 = \{48, 18, 1\}$ — *identical* in the deep layers (which are what a larger arrangement inherits) and behind only in depth-1, the layer the larger arrangement recuts anyway. The perfect triple pays for those four disposable regions with its last tunable degree of freedom. A good building block is deep-saturated, shallow-detuned, and keeps its knob.

One observation here seeds a major theme. There are two *different* three-cube arrangements that both reach the maximum of 67, and they achieve it by different geometry:

- The **octahedral** three-cube compound (three cubes each turned 45° about a different coordinate axis), which uses $\sqrt{2}$.
- Any three cubes from the golden five, which uses $\sqrt{5}$.

They differ in *how their faces line up*, a distinction ("edges versus corners") made precise in Section 6.

4. The six-cube search and the record chain

For six cubes the maximum is not known. Two facts shape the search.

First, **random angles are not enough**. If you pick six rotations at random, you land in a "generic" arrangement — no special alignments — and these top out around 635 bounded pieces no matter how many millions you try. The record-setting arrangements are *special*: they sit on lower-dimensional **walls** in the space of configurations, sets where some cubes line up exactly. Walls are vanishingly rare (measure zero), so random search never hits them. Every record beyond 635 came from deliberately *constructing* an alignment, not from sampling.

Second, **the good walls are symmetric**. An arrangement that looks the same after some rotation (has a symmetry) forces its cubes into related families. The current chain of records, and how each was found:

pieces	the arrangement	how it was found
635	a balanced generic-looking config	278,000-config random campaign + local search
655	two pairs of cubes at a special 60° body-diagonal angle	built the alignment by hand
681	the golden five + a sixth cube on a shared axis	search among golden ($\sqrt{5}$) configurations

699	two three-cube groups sharing one diagonal axis	overlaying two symmetric triples
705	same family, searched more thoroughly	symmetry search with better coverage
717	four cubes in a symmetric cluster + two more	symmetry search
723	three cubes about a diagonal + three "free" cubes	shared-axis "intersection" family

723 is the current record. Its depth profile is {1: 210, 2: 216, 3: 164, 4: 96, 5: 36, 6: 1} (summing to 723). Its six cubes, as integer quaternions:

4,1,1,-1 ; 3,3,7,3 ; 5,-1,-5,-5 ; 2,1,1,1 ; 1,1,1,1 ; 5,2,2,2

The searches that produced these fall into four kinds, each fixing a blind spot of the previous:

1. **Mass random campaign.** Counting ~278,000 random configurations exactly mapped the generic behaviour and disproved some early guesses about the maximum (an earlier "record" of 623 turned out not even to be a local best).
2. **Hand-built walls.** Deliberately imposing a known alignment (the golden compound, or a special pair angle) and searching the leftover freedom — this found 655 and 681.
3. **Symmetry catalogue.** Every symmetric arrangement is a union of symmetric families of cubes ("orbits" of a rotation group). The rotation groups are a known finite list (cyclic, dihedral, tetrahedral, octahedral, icosahedral), so the symmetric arrangements form a *finite catalogue* one can walk through systematically. A first pass under-searched (its parameter grid was too coarse and missed its own best case), but re-run properly it produced 705, 717, and 723, and showed no other symmetry type beats them.
4. **Algebraic search** (Section 8): solve for the exact alignments directly with symbolic algebra instead of approaching them by trial.

5. Patterns that hold across every arrangement

Several exact regularities appear everywhere we look:

- **All the depth-layer caps obey one formula.** The ceiling for the layer l steps up from the deepest is $C(l, n) = (12l-6) \cdot n - 2(l^2-1)$: the second-deepest layer is capped at $6n$, the next at $18n-6$, then $30n-16$, then $42n-30$ — and the outermost layer at $10n^2-14n$. Across roughly a million exactly-counted arrangements (two to seven cubes), every testable cap is *attained exactly and never exceeded*. The golden compounds attain

precisely the outermost-layer cap (48, 104, 180 at $n = 3, 4, 5$); the records attain the deep caps; no arrangement attains all caps at once (that is the frustration, and summing the caps gives an upper bound on the total — 801 at six cubes, against the record 723). The same formula is confirmed by a completely independent measurement: the vertex counts of the sphere diagrams of Section 6 match its prediction exactly. Proving it is now the project's central theory goal.

- **Deep layers are capped — and the second-deepest cap is exactly $6n$.** The depth-3, depth-4, depth-5 counts of a six-cube arrangement never exceed 164, 102, 36, on any arrangement tried. (Depth-6 is always 1, provably — the innermost region is one convex blob.) The last of these, depth-5 ≤ 36 , turns out to be one case of a clean law across all sizes: the **second-deepest layer, depth- $(n-1)$, never exceeds $6n$** , and equals $6n$ generically — verified for every size from two to six cubes (12, 18, 24, 30, 36). This unifies five separately-observed ceilings into one statement with a short proof sketch: the "which cube is innermost" pattern on the direction-sphere (Section 6) is anchored at each cube's six face-centre directions, giving at most six regions per cube, hence $\leq 6n$. Making that rigorous reduces to a single lemma about how the face-normals' directions can overlap; the lemma holds in every case sampled but is not yet proven in general (Section 10). The $6n$ cap is also **hereditary**: removing any cube from a record drops depth- $(n-1)$ from $6n$ exactly to $6(n-1)$ in the smaller arrangement.
- **Deep layers are "quantized," shallow layers grow — and records trade between them.** This is the central structural fact, visible across ~457,000 counted arrangements. As the total climbs, the deep layers fill to their caps *in order, deepest first, then freeze*: depth-5 reaches 36 (around total 475), then depth-4 reaches ~102 (around 525), then depth-3 reaches ~164 (around 600). The reason is geometric (Section 6): a deep layer either realizes its one *generic* value (the cap) or a smaller degenerate value — it cannot exceed the cap, so it is effectively quantized. The shallow layers (depth-1, depth-2), by contrast, have no cap and keep growing with the arrangement's complexity. So once the deep layers are pinned, every further gain in the total comes from the shallow layers alone (depth-1 climbs from ~120 to over 200; depth-2 from ~195 to ~215). Two kinds of trade-off result:
 - *At a fixed total*, regions shuffle between adjacent layers with the total conserved (in one family depth-2 + depth-3 stays exactly 368).
 - *To raise the total*, since the deep layers can't grow, the shallow layers do; and near the ceiling one can *give back* a little deep count to recover shallow count. The record's total 723 is reached along a whole ridge of profiles — with depth-4 at its cap of 102, or dropped to 96 with depth-1+depth-2 correspondingly higher — the trade being about one-for-one. Sacrificing deep count *beyond* that ridge loses (the shallow recovery falls short). Depth-1 has been pushed as high as 224 in some arrangements, so the shallow ceiling is high; the deep caps, not depth-1, are the true limit — but a search over 20,000 configurations found this trade-off surface *peaks* at 723, evidence (not proof) that the record is a genuine optimum of the surface, not just a point on it.

- This reframes the whole optimization: the goal is not to maximize any single quantity but to find the best point on this trade-off surface — which deep layers to sacrifice, and by how much, to unlock the most shallow gain. Mapping that surface (the best shallow count reachable at each deep-layer profile) is the current line of attack on 723.
- **A parity rule.** For a generic arrangement of n cubes, the total is always congruent to $2n-1$ modulo 4. (Reason: each cube is symmetric through the centre, so pieces pair up under the point reflection $x \rightarrow -x$, forcing the count into a fixed residue.) Verified for $n = 4 \dots 7$. Highly symmetric walls occasionally jump to the other allowed residue.
- **Richer sub-parts, higher total.** How many regions each individual cube (or pair) contributes correlates *positively* with the total (correlation $\approx +0.64$ for single cubes). The record is both rich and balanced — in the 723 configuration every one of the six cubes individually contributes exactly 30 depth-1 regions.

6. The geometry of the maxima: edges versus corners

The maxima are organized by **incidence** — extra coincidences where more face-planes than usual pass through a single point. In a generic arrangement, planes meet three at a time at ordinary corners of the pattern. A *concurrence* is a point where more than three meet. Two kinds matter, and they are exactly the "edges versus corners" seen back in the three-cube case:

- **Edge concurrence:** a point where **4 planes** meet, made of **2 planes from one cube and 2 from another**. Two planes of a cube meet along one of its *edges*, so this is an edge of one cube crossing an edge of another. Such points sit at distance $\sqrt{2}$ from the centre (cube edge-midpoint distance). This is how the *octahedral* three-cube maximum is built.
- **Corner concurrence:** a point where **6 or 9 planes** meet, made of **3 + 3 or 3 + 3 + 3**. Three planes of a cube meet at one of its *corners*, so this is two (or three) cubes whose corners land on the same point, at distance $\sqrt{3}$ from the centre. This is how the *dodecahedral/goldenthree-cube* maximum is built.

A clean fact ties this to the whole search: "several cubes share a corner" is exactly the same as "those cubes differ by rotations about the axis through that corner." So corner-sharing *is* the shared-axis symmetry — which is why the shared-axis families are where the records live. The record 723 has two 9-fold corner concurrences (three cubes meeting at a corner, twice), and is thus *corner-dominated*, though it also contains about 180 lesser edge concurrences. (There are never any *line* concurrences — three planes sharing a whole line — because that would need three parallel-enough normals, which no arrangement realizes.)

There is a sweet spot, not "more is better." One might guess that forcing even more planes through a point helps. It does not: forcing four cubes to share one corner (a 12-fold concurrence) yields only 393 pieces — far below the record. Over-concentrating the coincidence merges away too many regions. The record's 9-fold concurrence is near the optimum.

A question raised by this picture — now largely answered. Every six-cube record is corner-dominated. Could there be a six-cube maximum built mainly on *edge* concurrences

instead? A dedicated search (about 4,600 exact configurations across four strategies, including exact $\sqrt{2}$ arrangements) says no. The best edge-dominated arrangement reaches only 691 — an edge-*pure* configuration of two octahedral three-cube compounds — which is 32 short of 723, and the entire gap is in depth-1 (174 versus 210). Moreover, edge-richness *anti-correlates* with the total (rank correlation ≈ -0.58 across the structured families): the more edge concurrences you force, the *fewer* pieces you get, and an edge-maximizing search drives the total down. So corner concurrences are genuinely the stronger ingredient, and substituting edges for corners does not reach the record. This is strong evidence, not a proof — an unsampled region of configuration space, or a wall needing numbers beyond $\sqrt{2}$, is not excluded.

Toward proving the caps. There is a reformulation that turns the whole problem into geometry on a sphere. Point in every direction $\hat{u}$ from the centre; the cube k extends out to distance $t_{\square}(\hat{u}) = 1 / \|\mathbf{R}_{\square}^T \hat{u}\|_{\infty}$ in that direction. The regions of a given depth correspond exactly to connected zones on the direction-sphere where a particular set of cubes are the "innermost." The boundaries of these zones are arcs of great circles, and counting the zones becomes an exercise in counting cells of a great-circle arrangement — for which there is an exact vertex/edge/cell census. That census comes out to fixed numbers (36, 102, 164 for depths 5, 4, 3) for generic arrangements, matching the observed caps; making that rigorous, plus showing that special alignments can only *merge* zones (never create extra ones), would prove the caps. The first half is established; the second half is the remaining gap.

7. The dihedral family: a closed-form bridge between the two 67s

Section 6 showed the $n=3$ maximum (67) is reached in two geometrically different ways — an *edge-dominated* octahedral compound ($\sqrt{2}$) and a *corner-dominated* golden compound ($\sqrt{5}$). This section describes a one-parameter family of three-cube compounds, found afterward, that contains **both** 67s as special points and threads exact face-plane coincidences through every point in between — answering, in closed form, what kind of continuous path connects them. It was found by acting on a concrete observation made while looking at the interactive viewer: the near-miss edge crossings along the midpoint of an earlier, cruder attempt at such a slide (Postscript 9 of the ledger) looked like they were sitting in a plane perpendicular to the direction (1,1,1) — and that observation turned out to identify the whole family.

The family. Take the cube $[-1,1]^3$ and an axis $\mathbf{n}(\psi) = (\sin \psi, \cos \psi, 0)$ that lies *in* one of the cube's own face planes, through the centre. Rotating the cube by $+120^\circ$ and by -120° (i.e. $+240^\circ$) about $\mathbf{n}(\psi)$ gives two more cubes; together with the original (identity) cube, this is a compound of three cubes, symmetric under 120° rotation about $\mathbf{n}(\psi)$ (a C_3 *orbit*). Every member of the family has an extra symmetry too: a 180° rotation about the cube's own horizontal face axis maps the whole compound to itself (dihedral symmetry D_3), which is where the name comes from. The parameter ψ ranges over $[0^\circ, 90^\circ]$; two exact symmetries cut this down further:

- **Mirror symmetry.** Swapping the x- and y-coordinates carries the configuration at ψ into the configuration at $90^\circ - \psi$ with all three phases negated — an isometry, so the two have

identical region counts and depth profiles. (Proof: the coordinate swap P has $\det = -1$ and sends $n(\psi)$ to $n(90^\circ - \psi)$; conjugating a rotation by an improper orthogonal map reverses its sense, giving exactly the $\psi \leftrightarrow 90^\circ - \psi$, phase-negation correspondence.)

- **90° periodicity.** ψ and $\psi + 90^\circ$ give the identical compound (not just a congruent one) — rotating the *seed frame* by 90° about its own third axis is itself a symmetry of the cube. So the family's true, non- redundant parameter range is $\psi \in [0^\circ, 45^\circ]$.

The coincidence identity. For every ψ (not just special values) and every choice of phases, the corresponding edges of any two cubes in the family — matched up by "same class" (there are three classes, one per coordinate axis of the seed cube) — are guaranteed to lie in a common plane, and non-parallel lines in a common plane always meet. So the family has built-in exact edge concurrences everywhere, not only at isolated points. (One class is elementary: all three cubes' relevant edges lie at one of two fixed heights measured along $(1,1,1)$, and any two edges at the same height share a plane. The other two classes follow from a short vector computation in the family's natural frame. This is now a proved theorem, not just a numerical observation — see below.) What *does* vary with ψ is only whether a given coincidence lands inside the finite edge segment (making it a real crossing of the compound) or outside it (making it geometrically real but physically absent) — that is what produces the plateaus and spikes described next.

Both 67s are members of this family.

- The **octahedral** 67 sits at $\psi = \arcsin(1/\sqrt{3}) \approx 35.264^\circ$, axis $n \propto (1, \sqrt{2}, 0)$. Its pairwise congruence invariant matches $1/2 + \sqrt{2}$, as expected from the $\mathbb{Q}(\sqrt{2})$ witness of Section 6.
- The **golden** 67 sits where $\tan \psi = \phi^2$, $\phi = (1 + \sqrt{5})/2$ the golden ratio — equivalently $\sin \psi = \phi/\sqrt{3}$, $\cos \psi = 1/(\phi\sqrt{3})$, a consistency that boils down to the identity $\phi^2 + \phi^{-2} = 3$. Its pairwise invariant is $3\phi/2$.
- A **new exactly-certified compound** sits at $\psi = 45^\circ$ exactly — the face-diagonal axis $(1,1,0)/\sqrt{2}$. Its rotation matrix has entries in the field $\mathbb{Q}(\sqrt{6})$ (a new "field clone" engine, `q6_count.py`, was built to count it, following the same pattern as the project's other $\mathbb{Q}(\sqrt{d})$ engines). Its exact region count is **49**, depth profile $\{1: 30, 2: 18, 3: 1\}$ — verified on a single engine so far, not yet cross-checked by a second independent counter.

The staircase. Sweeping ψ across $[0^\circ, 90^\circ]$ with exact $\mathbb{Q}(\sqrt{3})$ arithmetic (rational sin/cos pairs from Pythagorean triples give rotations in $\mathbb{Q}(\sqrt{3})$, which a new engine `q3_count.py` — again a field clone of the project's validated counting code — can count exactly), the region count forms a symmetric staircase around $\psi = 45^\circ$:

ψ range	count	depth profile
$[0^\circ, \approx 9.6^\circ]$, incl. shared-axis endpoint	25	$\{12, 12, 1\}$

$(\approx 9.6^\circ, \approx 10.9^\circ)$	31	{18, 12, 1}
$(\approx 10.9^\circ, 20.905^\circ)$	43	{24, 18, 1}
35.264° (octahedral)	67	{48, 18, 1}
$(20.905^\circ, 69.095^\circ)$, central plateau	55	{36, 18, 1}
45° (face-diagonal, new)	49	{30, 18, 1}
69.095° (golden)	67	{48, 18, 1}
... mirrored on the other side, 90° (shared axis)	25	{12, 12, 1}

Two things stand out. First, the depth-3 count is always 1 and the depth-2 count is 18 across the whole central plateau (dropping to 12 only near the two shared-axis endpoints) — every bit of variation is in the depth-1 (outermost) layer, an especially clean instance of the "deep-structure-conserved, shallow layer is what varies" principle of Section 5. Second, and more surprising: **the face-diagonal point is a local *minimum*, not a local maximum** — its extra edge coincidences (6 more than the plateau) happen to *merge* regions rather than *create* new ones, while the extra coincidences at the octahedral and golden points (12 more than the plateau) *do* create new ones. This is the sharpest illustration yet, at $n=3$, of a theme from Section 6: what matters is not how many face-planes concur at a point but exactly how they concur — coincidence-richness cuts both ways.

Why the earlier slide had ghosts. The project's original attempt at a continuous path between the two 67s (Postscript 9 of the ledger) does connect them, but its interior points are *not* on this family surface — its seed direction has a small but nonzero component along (1,1,1) where the family requires exactly zero — so its near-miss crossings never quite close ("ghosts": a visible near-alignment with a small but nonzero gap). The dihedral family threads the same two endpoints through a path of *exact* coincidences the entire way; the ghosts of the old slide are precisely the cost of stepping off that surface.

The persistent 18-core. Restricting to the interval between the two golden copies, $\psi \in (20.905^\circ, 69.095^\circ)$, the *set* of interior edge crossings — not just their count — is exactly the same 18 pairs of edges for the whole open interval. The counts of 30 (at octahedral), 24 (at the face-diagonal point), and 30 (at mirror-octahedral, $\psi = 90^\circ - 35.264^\circ = 54.736^\circ$) are momentary spikes: +12, +6, +12 *extra* coincidences that exist only exactly at those isolated points and never open a gap in the core 18. Arriving at either golden endpoint, nothing in the core breaks: 6 of the 18 stay interior (at segment position $t = \pm 1/\varphi^3$ along the edge — another golden-ratio appearance), and the other 12 land exactly on cube corners, becoming golden's own

corner-coincidence structure. So the entire drag from octahedral to golden — through the face-diagonal point — can be made while keeping all 18 core concurrences continuously intact; the previous belief that the slide had two more unnamed transition points near 21° and 69° was a misreading of where the near-miss "ghost bands" begin and end, not real changes in the coincidence set. The exact transition points are only 20.905° , 45° , and 69.095° .

Chasing more than 18. Can a path be found that preserves *more* than the 18-core the whole way from octahedral to golden? A generalization of the family (n cubes on a common axis, arbitrary phases, one shared tilt ψ) has a *pair-curve identity*: the four "extra" coincidences of the octahedral pair all vanish along one single curve in (phase, ψ)-space through the octahedral point, not only at the octahedral point itself. Riding this curve with a three-cube chain of phases achieves **26** simultaneous concurrences over a sub-range of ψ (35.264° up to about 44.5°) — a genuine improvement over 18, but only locally: the curve's valid segment closes before reaching golden, and a dedicated search for a way past this wall (tracing all vertex-adjacent relabelings at the point where the curve exits, and separately tracing golden's own analogous extra-coincidence curve backwards) found no route that reaches golden while carrying more than 18 physical concurrence points. The obstruction has a specific location: both the octahedral-side curve and the golden-side curve pass close to $\psi = 45^\circ$, but at phase values roughly 70° apart, and neither curve's far branch swings around to link them. So **18 stands as the best confirmed lower bound for an end-to-end path**, with a described local obstruction — not a proof that 18 is a ceiling.

Four theorems. Since this family was found, four of its properties have been formally proved (not just checked numerically to machine precision):

1. **Mirror symmetry** (stated above) — proved for every phase tuple and every n , not just $n=3$.
2. **90° periodicity** (stated above) — likewise general.
3. **The coincidence identity** — every same-class edge pair of any two family members is exactly coplanar, for every ψ and every phase tuple, proved by direct computation in the family's natural frame (one class is elementary; a second follows from a short vector identity; the third follows from the first two via the periodicity theorem). What is *not* yet proved is exactly which ψ -ranges keep a given coincidence inside the finite edge segment — that is a concrete, well-posed algebra problem (finitely many polynomial sign conditions), not yet carried out.
4. **A rational-invariant obstruction.** If every cube in a configuration has rational (integer-quaternion) coordinates, a certain pairwise congruence invariant is forced to be rational too. Both known 67 witnesses have irrational invariants ($1/2+\sqrt{2}$ and $3\phi/2$), so **no rational configuration can be congruent to either** — the two known 67s are provably not reachable by any rational arrangement. If they are also the *only* three-cube maximizers up to congruence (believed, supported by their isolation and by every search so far, but not proved), this yields a striking corollary: **$n=3$ is the unique irrational level of the whole record tower** — $n=2$'s record (13) is rational, and so is every current record at $n \geq 4$, while $n=3$'s maximum provably cannot be.

Extending to n cubes, and what it reveals about the records. The family generalizes past $n=3$: n cubes with independent phases about the same axis and the same shared tilt ψ , with the pairwise relative rotation between any two members depending only on their phase difference and ψ (a closed Rodrigues-rotation form). At Pythagorean ψ this makes every member an **integer-quaternion** configuration, countable directly by the project's fast C++ engine — turning the family, for the first time, into something the record-search machinery can search exhaustively and exactly. Two results:

- **As a search space by itself, the family loses ground as n grows.** The best purely-single-axis family members found (an exact sweep of over 9,000 configurations at $n=4,5,6$) reach only 175, 335, 615 — falling short of the true records 183, 393, 723 by 8, 58, and 108 respectively, a deficit that *widens* with n . (This measures the family's Pythagorean-sampled plateau, a lower bound on the true family's supremum — the family's own irrational spikes, like the two 67s at $n=3$, are invisible to a rational sweep and could in principle raise these numbers, an open question.)
- **But every current record is built almost entirely out of family pairs.** Checking every pair of cubes in each record against the family's own membership test: **all 6 pairs** of the 183 record, **all 10 pairs** of the 393 record, and **12 of the 15 pairs** of the 723 record are in family position (some shared axis and tilt — not necessarily the same axis for every pair). The 723 record's structure resolves exactly: five of its cubes form a complete family clique (this is exactly the embedded 393 record, consistent with the nesting property of Section 9), and the sixth cube is family-linked to two of the five and generic against the rest. So the records are **gluings of family cliques living on different axes**, not members of one common- axis family — which is exactly why a single-axis sweep falls behind: it searches a strict subset of the structure the records actually use. This reframes the record search itself: instead of a raw search over all rotations, it becomes a search over how many family cliques to use, on which axes, and how to glue them — a search that is still open and ongoing.
- The deep layers of the best family members match the corresponding record's deep layers *exactly* at every n tested (e.g. the $n=4$ family best has the same depth-3 and depth-4 counts as the 183 record, with its entire 8-point deficit sitting in the depth-2 layer alone) — the clearest demonstration yet that a good building block is deep-saturated and only ever detunes the shallow layers (Section 3's frustration principle, now traced through an entire new family at every size).

Files: `q3_count.py`, `q6_count.py` (the two new field-clone engines), `nfamilys_common.py/nfamilys_sweep.py/nfamilys_q3_records.py` (the n -cube generalization and record-pair analysis), and `dihedral1_scratch/` (exploration and verification scripts). The formal proofs are in `C45_notes.md`, section 12; the numerical exploration is `six_cube_search_results.md`, Postscripts 25 and 26.

8. Solving for the alignments algebraically

The searches above *approach* the special walls numerically. A more direct approach *solves* for them. Because a rotation about a fixed axis can be written with a single rational parameter (keeping all coordinates as exact fractions), the condition "this face passes exactly through that point" becomes a polynomial equation, and the exact solutions can be found with computer algebra (Gröbner bases and equation solving):

- **One-parameter sweep.** Sliding one symmetric group of cubes along its shared axis and solving where a new coincidence forms yields the finite list of exact "special angles" (126 of them along one such slide) — the exact walls, found without any grid. Counting them confirmed 723 is a genuine local maximum along that slide.
- **Multi-constraint solve.** Requiring a new cube to lock onto two existing cubes' corners at once is a small system of polynomial equations; solving it exactly produces configurations a numerical grid would never land on. This found a family reaching 689 with a record-high depth-1 of 224 (but a lower total, because the deep layers fell) — quantifying the same shallow-versus-deep trade-off, and confirming the deep caps as the true bottleneck.

Neither has beaten 723, but together they map the relationship between coincidence structure and piece-count exactly, and can reach exact algebraic (even irrational) configurations that trial-and-error cannot.

- **The rational-tangent slice.** Dissecting the records showed their internal cliques sit at tilts with rational *tangent* but irrational *sine* ($\tan \psi = 2/3$ and the like) — a slice of angles every earlier sweep, which used rational-sine tilts, had missed entirely. At such a tilt a phase step keeps the whole configuration rational exactly when its parameters are a rational point on a conic $c^2 + d \cdot s'^2 = 1$, and those points have a one-parameter formula closed under addition, so the integer-only engine can walk the slice. The 393 record's four-cube clique is exactly such a conic chain. Sweeping the slice broke an apparent "constant-8" deficit (an earlier gluing search fell 8 short of the record at four, five, and six cubes alike): a single-axis five-chain with non-uniform conic phases reaches **387** at five cubes, cutting the deficit to 6. It also revealed the six-cube record to be a *plateau* — 27-plus non-congruent sixth cubes all completing to exactly 723, in four depth profiles that trade middle-layer count at a fixed rate.
- **The n=4 resonance, resolved negative.** Writing down and exactly solving the conditions for extra cross-alignments at four cubes, then counting every solution, shows every such resonance in the reachable number fields *merges* rather than creates: the best counts 151, below even the family's rational plateau of 175. The +12 that irrationality buys at three cubes does not recur at four. (A few higher-degree candidates remain uncounted for want of an exact-sign routine — one in the record's own number field — the single honest caveat.)

From search to branch-and-prune. The structural findings collapse the search space into something almost combinatorial. Since a configuration's worth is carried by its *blueprint* — how the cubes partition into shared-axis clusters, which pairs are rigid best-pairs versus tunable

9-pairs, which cubes are free — the search becomes: enumerate the finite catalog of blueprints, *prune* the ones that provably can't win (a triangle of rigid best-pairs forces the golden wall, whose extensions are known to lose; geometrically inconsistent labelings never existed), and optimize the surviving blueprints over their continuous knobs. The prize is a qualitatively stronger conclusion than any search so far: "no blueprint beats 723," exhaustive at the skeleton level, rather than "we didn't find one." This search has now run to completion: 391 raw blueprints collapse to 67 canonical skeletons (plus two pruned with documented reasons — the golden wall, whose extensions provably lose, and the polyhedral-forcing family, tested and dominated); all 67 were knob-optimized (over 83,000 exact counts) and none beats 723. What would upgrade it from branch-and-*prune* to true branch-and-*bound* is one missing theorem — a **deficit-propagation lemma**: a guarantee that if a partial assembly's sub-compounds fall short of their deep caps by some amount, the completed arrangement's total is capped below the global bound by a corresponding amount. The empirical envelope for that lemma has now been measured (every five-cube sub-compound of 532 arrangements spanning the whole spectrum, counted exactly), and it is sharp on both fronts: adding a sixth cube never gained more than 336 over the best five-cube part — so any arrangement beating 723 must *contain* a five-cube arrangement of at least 388, a class that only the record's own family is known to occupy — and any arrangement whose parts miss their deep caps is capped more than 150 below the record. Both statements are measured envelopes awaiting proof, but together they nearly corner 723: the search for a better six-cube arrangement reduces to the search for a new near-record five-cube one.

9. More than six cubes, and the record tower

The counting method works for any n . Best arrangements found so far (lower bounds on the true maxima):

cubes n	2	3	4	5	6	7	8
best bounded pieces	13	67	183+	393+	723	1207+	1879+

Several of these numbers are *corrections*: the golden compound gives 177 at four cubes and 351 at five, and both were long taken as the answers — but a rational four-cube arrangement reaches 183 and a rational five-cube one reaches 393. Golden is the best *symmetric* configuration, not the maximum. This raised the question of whether even the small cases are safe; thorough search in their low-dimensional spaces **confirms 13 and 67** (nothing beats them), and confidence rightly runs opposite to size: near-certain at $n=2$, strong at $n=3$, down to "best found so far" at $n=6,7$.

The records form a tower, and it is generative. The best six-cube arrangement *contains* the best four-cube one and a five-cube sub-compound (393) that beats the old five-cube record; adjacent sizes differ by adding or removing one cube. This is not just descriptive — it *builds* records: taking the 723 six-cube record and adding a single seventh cube, over a few hundred

orientations and with no fine-tuning, already reaches **1207** at seven cubes, beating a 50,000-arrangement random campaign (which topped out at 1085). So the recipe "take the best $(n-1)$ -cube arrangement and add one cube well" outperforms searching from scratch.

The tower is organized by a shared 3-fold symmetry. Each golden-beating record keeps exactly a single 3-fold rotation axis (a symmetry group of order 3) — dropping all the way down from golden's order-60 icosahedral symmetry, but never becoming fully asymmetric. That surviving axis is the corner-sharing structure of Section 6, and it organizes the tower: the cubes split into one 3-fold orbit plus a few axis-fixed cubes, the orbit cubes are interchangeable (removing any gives the same result), and it is the *axis-fixed* cubes that are added and removed to climb between sizes. The parity rule, the $6n$ second-deepest cap, and the shallow-gain / deep-cap structure all persist as n grows.

The 3-fold orbit is a dihedral-family clique. Section 7 sharpens this picture: the surviving 3-fold orbit at every size is exactly a clique of the dihedral family (Section 7), and the axis-fixed cubes attach to it in family position too, wherever tested — the records are gluings of family cliques on different axes, not one common-axis structure. That reframing is what suggests the two-clique gluing search of the open problems below.

The six-cube record is a plateau, not a point. Fixing the record's first five cubes and sweeping a sixth over thousands of integer orientations turns up at least 27 distinct completions all counting exactly 723, in four different depth profiles related by an exact middle-layer exchange (depth-2 and depth-4 rise by 2 as depth-3 falls by 4, holding depth-1, depth-5, depth-6 fixed). The record *value* is one number; "the 723 arrangement" is at least four non-congruent arrangements — the same shallow-for-deep conservation seen throughout, now operating exactly at the summit.

10. Open problems

1. **Prove the second-deepest cap, $\text{depth}-(n-1) \leq 6n$ — now with a draft proof.** This reduces (Section 5) to one lemma: the direction-sphere function "distance to the nearest face" has local minima *only* at the $6n$ face-centre directions, with no extra "shoulder" (or "parasite") minima. The two-normal case was long provable; the open crux was whether three or more face-normals could conspire to make an extra minimum. A short argument now appears to close it for every n , built from three facts already in hand: at a point where several faces tie for closest, their spherical gradients have *equal length*; the tie curves are arcs of great circles, and distinct great circles are never tangent; and a linear-algebra alternative (Jordan's theorem) forces the equal-length gradients either to admit a strictly-improving direction that the great-circle geometry keeps inside the cell, or to be exactly equal — meaning two genuinely parallel faces, a degeneracy that removes itself. The write-up (PROOF_L1b.md) is filed as a draft pending one adversarial read; if it holds it proves this cap for all n and, with the convexity bound $\text{depth}-2 \leq 1$, settles **max = 13 at two cubes**, the first complete maximum theorem. (The deeper caps $\text{depth}-3 \leq 164$ and $\text{depth}-4 \leq 102$ at $n=6$ are separate, harder counting statements.)

2. **"max = 67 at three cubes" — PROVED (2026-07-21).** $\text{depth-2} \leq 18$, $\text{depth-3} \leq 1$, and the shallow bound $\text{depth-1} \leq 48$ are all now in hand, so **max(3) = 67 for all three concentric convex ≤ 6 -facet cells meeting pairwise transversally**, cubes included. $\text{depth-1} \leq 48$ came via the top diagram's vertex weight $\text{Sum}(\text{deg}-2) \leq 92 = 32$ (triple points) + 60 (contacts): contacts bounded by Euler on the three pairwise intersection polytopes ($\text{Sum}(\text{deg}-2) = 2F-4$, ≤ 20 each), triples by the bottom-diagram Euler, and the degenerate triple points — where the tempting " $\text{farthest-diagram} \leq \text{nearest-diagram}$ " is *false* — by a two-budget local inequality charging each triple point to the bottom diagram and the pairwise polytopes at once. Full proof: `PROOF_STEP_T.md` and `PROOF_67.md`; story in `PROOF_NARRATIVE.md`. Only residual caveat: the pre-existing pairwise-tangency degeneracy (two cells sharing a boundary tangentially), not a triple-point issue.
3. **Beat 723 for six cubes, or prove it is the maximum.** In the trade-off language of Section 5, this means finding the best point on the deep-sacrifice surface: since the deep layers are capped and the shallow layers are not, the question is exactly *how much* deep count to give back to unlock the most shallow count. 723 sacrifices depth-4 alone; whether sacrificing depth-3 and depth-4 together (or another combination) nets higher is open and is the active line of attack. Depth-1 can already reach 224, so there is shallow headroom.
4. **Can non-concentric or unequal-sized cubes do better?** Everything above keeps the cubes centred at a common point and equal in size. Both restrictions can be relaxed, and each only *adds* freedom, so neither maximum can be below the concentric-equal one — the question is whether either is strictly larger. Two experiments (with a counter built for the purpose, validated to reproduce the concentric-equal counts exactly): **off-centring the record strictly hurts** — every translation lowers the total (best 706, then monotonically worse), so concentric is at least a local optimum. **Unequal sizes are so far untested at the record**, because 723 is so symmetric that resizing any cube by a rational factor creates an exact face-coincidence the (non-robust) counter cannot evaluate; settling it needs a degeneracy-tolerant counter. Note that resizing *preserves* the central symmetry that off-centring destroys, so it is the gentler perturbation and the more plausible of the two to help — but there is no proof either way.
5. **Settle the edge-versus-corner question:** can an edge-dominated arrangement ever match a corner-dominated one at the maximum? Current evidence says no — edge-dominated arrangements top out at 691 and edge-richness anti-correlates with the total — but this is evidence from a finite search, not a proof. A related, sharper question raised by Section 7: can any path in the dihedral family carry more than 18 physical edge concurrences continuously from the octahedral 67 to the golden 67? A dedicated search found a specific local obstruction (two different "extra-coincidence" curves that both graze $\psi=45^\circ$ but stay about 70° apart in phase) and no rescue past it — 18 is a confirmed lower bound with a described obstruction, not a proven ceiling.
6. **Special number-field walls:** whether any arrangement needing $\sqrt{2}$, $\sqrt{3}$, or a combination beats the best plain-fraction arrangement. (The golden $\sqrt{5}$ wall reaches 681; the overall record 723 is a plain-fraction arrangement.) Section 7 adds one more exactly-certified data point in this vein — the $\sqrt{6}$ face-diagonal compound at 49 —

well short of the record and, notably, a local *minimum* of its own family rather than a maximum.

7. **Growth with n** and the persistence of the structural laws at scale.
8. **The clique-gluing search (carried out; no record beaten).** Section 7 showed every current record is a gluing of dihedral-family cliques on different shared axes. Reframing the search this way — choosing cliques, axes, tilts, phases — and running it over the rational-tangent slice (Section 8) reached **387** at five cubes, breaking a spurious "constant-8 deficit" from an earlier, more restricted gluing search, but did not pass a record; it also proved the six-cube record a plateau (Section 9). Whether the reframed space contains anything beating 723 remains open, but it now looks like a landscape whose peaks are the known records rather than a space hiding a larger one.
9. **The $n=4$ resonance, resolved negative.** The two $n=3$ maximizers sit at irrational, algebraically special points of the dihedral family. The analogous question at four cubes — does an irrational resonance beat the family's rational plateau of 175? — has now been solved symbolically: every four-cube resonance in the reachable number fields *merges* regions, the best counting 151. Irrationality, which buys +12 at three cubes, does not pay at four. Combined with the (conditional) uniqueness of the two 67s, this makes **three cubes the sole irrational rung** of the tower the best-supported conjecture in the project. One escape hatch stays open: a short list of higher-degree candidates left uncounted for want of an exact-sign routine, one of them in the record's own number field.
10. **A create-versus-merge criterion.** A new exact coincidence can raise the region count or lower it; " ± 1 region per coincidence" is exact for the pure interior-crossing events at three cubes and false everywhere else (it fails even in *sign* at four cubes). What holds universally, on every event tested, is that the entire change lands in the shallowest depth layer — because coincidences are exactly the vertices of the top diagram and never touch the deep-layer bottom diagram (Section 5's two-diagram picture, confirmed by the maxima's exact censuses). A closed criterion — does an added coincidence's vertex weight exceed the arc structure it consumes? — is now a one-diagram bookkeeping question, but is not yet written down.

11. The code, briefly

The exact counter (`cube_regions`, C++), its slower cross-check (`certify_six.py`), the search drivers, the symmetry catalogue, and the algebraic-search scripts (which drive the computer-algebra system `wolframscript`) all live in this directory; a companion `README.md` maps each file and gives runnable commands, and `six_cube_search_results.md` is the dated primary record of every result. Two further exact engines support Section 7's dihedral family: `q3_count.py` (field $\mathbb{Q}(\sqrt{3})$, for Pythagorean-angle family members) and `q6_count.py` (field $\mathbb{Q}(\sqrt{6})$, for the face-diagonal point), both built as field-constant clones of the project's earlier validated $\mathbb{Q}(\sqrt{2})$ counter. A single configuration can be counted directly, for example:

cube_regions --quats '4,1,1,-1;3,3,7,3;5,-1,-5,-5;2,1,1,1;1,1,1,1;5,2,2,2'
→ 723 bounded regions

Every record quoted here has been re-verified by an independent counter.

Interactive depth explorer. A browser-based tool renders any configuration (paste six quaternions; prefilled with the record) two ways: a draggable cross-section whose every pixel is coloured by depth — sweep the cutting plane through the compound and watch the shallow shell and white-hot core appear — and a rotatable 3D point cloud of the same depth structure. Depth and per-cube filters isolate any layer or any cube's contribution to cut clutter; a toggle recolours cells by *which* cubes contain them (containment label) rather than how many. Since the last update the viewer gained several features, all under the same published URL:

- **Opaque mode.** A toggle renders the compound's faces as solid, shaded, depth- or containment-coloured polygons (each cube face split by every other selected cube's face-planes into its true convex pieces, painter's-algorithm sorted) instead of a point cloud — concretely showing the region boundaries as surfaces, not dust.
- **The dihedral-family slider**, which replaces the old, ghost-gapped "67 ↔ 67 slide" of Postscript 9 with a slider along the family of Section 7: a ψ readout and drag control from 0° to 90° , named tick marks (including the newly-added mirror-golden point), a live ghost (near-miss) counter, and a "maintain concurrences" lock that clamps dragging to stay within a range where the crossing set — the core-18, specifically — is certified constant.
- **Split/merge surface highlighting**, which outlines exactly the opaque faces involved in whatever near-miss coincidences ("ghosts") exist at the current slider position — a live, ψ -dependent view of which surfaces are about to split apart or merge together.
- **Zoom** (mouse-wheel, threaded through the same scale factor as every other overlay) and **one-sided clipping** of the opaque surface against the cross-section plane, so the solid interior can be inspected without the near side occluding it.

[Source: <https://github.com/dmi-000/cubes/blob/main/PROJECT.md>]

Six Glass Cubes: A Week of Experimental Mathematics With a Team of AIs

Dean Inada, Chris Cole, Werner Couwenbergh, Claude Fable 5, Claude Sonnet, and Claude Haiku

An informal account, updated 2026-07-18. Self-contained, but every claim here has a paper trail: `six_cube_search_results.md` (the dated ledger, now Postscripts 1–31) is the primary record, `PROJECT.md` is the formal write-up, and `README.md` maps all the code. Anyone with access to a mid-tier coding model (Claude Sonnet or similar) and a laptop can reproduce everything — a "how to reproduce" section is at the end. And a note on authorship, since it matters for how to read this: this document, like the project's code, searches, and analysis, was written by an AI (Claude) working under human direction — see "The collaboration, honestly described" below for what that division of labor actually looked like.

The question

Take n identical cubes and stack them all at the same centre point, each rotated to its own angle, like a die photographed mid-tumble with every frame kept. Their faces slice space into cells. **How many bounded cells can n cubes make, and which set of angles is best?**

For one cube the answer is 1. For two it's 13, and already non-obvious. By six cubes the answer runs into the hundreds and — as far as we know — nobody had mapped it. We ended the week at **723** for six cubes, with a formula that seems to govern every size, three independent arguments that 723 is hard to beat, and a stack of conjectures that are begging for proofs.

One rule made the whole project possible: **every count is exact**. A configuration is given by n integer quaternions (four integers each — any four integers encode a rotation whose matrix has exactly-representable fractional entries), and the region count is computed with integer/ fraction arithmetic, no floating point in any decision. A reported number is a theorem about that configuration. This mattered more than we knew when we adopted it.

Act I: the false start that set the rules

The project began with the obvious approach: voxelize space, flood-fill, count components. It produced confident-looking numbers — one random six-cube configuration showed a "stable

plateau" around 1,340 regions across three grid resolutions. The exact count of that configuration turned out to be **567**. About 70% of the voxel "regions" were slivers thinner than any grid we could afford; meanwhile other real regions were being merged. Both failure directions were live at once, and resolution "convergence" proved nothing.

So the voxel ranking was thrown away entirely — a day's work — and the rule became: approximate methods may *suggest*, only exact counts *decide*. The exact pipeline is conceptually simple: start with a big box, cut it by all $6n$ face-planes into convex fragments (exact rational arithmetic), then merge fragments that touch across "phantom" walls — places where a fragment boundary lies on a face's infinite plane but outside the actual bounded square of the face. Union-find does the merging; a containment test labels each region by which cubes hold it. A C++17 version (cube_regions.cpp, no dependencies) counts a six-cube configuration in well under a second; a slower Python implementation exists purely to disagree with it, and never has (200-seed validation, zero mismatches). Every record below was verified on both engines.

Two exact anchors calibrate everything: six cubes fanned around a shared axis give exactly $(2 \cdot 6 - 1)^2 = 121$ (provable), and the classical compound of five cubes inscribed in a dodecahedron — "the golden compound", built on the golden ratio, coordinates in $\mathbb{Q}(\sqrt{5})$ — gives exactly **351**, with its sub-compounds at 1, 13, 67, 177. Hold those five numbers; four of them have a surprise coming.

Act II: the six-cube race

Random search hits a wall fast. About 278,000 random six-cube configurations, counted exactly: the best is ~635, and more sampling doesn't help, because the good configurations live on *walls* — measure-zero surfaces in configuration space where cubes align exactly. You never land on a wall by chance; you have to build it.

The record chain, each verified twice:

record	what it is	how found
635	best generic-looking config	278k campaign + local search
655	two pairs at an exact 60° body-diagonal relation	built by hand
681	golden five + a sixth cube on a shared axis	search in $\mathbb{Q}(\sqrt{5})$

699	two 3-fold triples sharing the (1,1,1) diagonal	overlying symmetric triples
705, 717	same families, searched properly	symmetry catalog, better seeds
723	a 3-cube cluster about (1,1,1) + three "free" cubes	shared-axis family

The 723 configuration, if you want to check it right now:

```
cube_regions --quats '4,1,1,-1;3,3,7,3;5,-1,-5,-5;2,1,1,1;1,1,1,1;5,2,2,2'
```

→ 723 bounded regions, depth profile {1:210, 2:216, 3:164, 4:96, 5:36, 6:1}

("Depth" = how many cubes contain a region; the profile is the count at each depth. It becomes the main character of this story.)

A meta-lesson from this phase: the 705/717 jump happened because the first systematic symmetry sweep was silently under-parameterized — its seed grid couldn't even represent the then-record 699, and it "found" that family capped at 399. The framework was validated, the coverage wasn't. Since then every search states its coverage explicitly and must re-derive the current record from its own machinery before its negatives are believed. That gate caught several other would-be phantom results.

Act III: the questions that broke the golden compounds

The best ideas in this project arrived as short human questions.

"The octahedral 3-cube maximum uses edge concurrences where the dodecahedral one uses corner concurrences." True, and it's exact: both three-cube maxima count 67, but one (three cubes at 45° about the three coordinate axes, needing $\sqrt{2}$) achieves it with 4-plane points where cube *edges cross*, at distance $\sqrt{2}$ from centre; the other (any three golden cubes, needing $\sqrt{5}$) with 6- and 9-plane points where cube *corners coincide*, at $\sqrt{3}$. A dedicated search then showed edge-dominated six-cube configurations top out at 691 and that edge-richness *anti-correlates* with the count (rank correlation ≈ -0.58): corners are simply the stronger ingredient. Also measured: concurrence has a sweet spot. The record's two 9-plane corner points (three cubes sharing a corner) are near-optimal; forcing a 12-plane point (four cubes through one corner) crashes a six-cube count into the 300s. More alignment is not better. A useful fact fell out for free: "k cubes share a corner" is *the same thing* as "k cubes related by rotations about the axis through that corner" — corner-sharing IS shared-axis symmetry, which is why all the record families have a 3-fold axis in them.

"So 177 is wrong?" (relayed from Chris Cole). The golden four-cube compound counts 177 and every reference treats the golden compounds as the natural optima. The $n=4$ search said otherwise: a plain-rational four-cube configuration reaches **183** (1,0,0,0; 0,5,3,2; 1,-4,-1,1; 1,1,-1,-4, profile {92, 66, 24, 1}). The technique that found it — climb to a local max, then make a *wide* multi-component jump and climb again, repeatedly — became the workhorse; plain greedy search stalls below the golden value and never sees 183.

"Does this call 351 into question?" Yes, and the answer was already in our pocket: drop one cube from the 723 record and the remaining five count **393** — beating the golden five-compound by 42. In fact *all six* five-cube subsets of 723 beat 351. Meanwhile $n=2$ (13) and $n=3$ (67) survived deliberate stress-testing; their configuration spaces are small enough to search almost densely. So the golden compounds are the best *symmetric* configurations, and the true maxima are asymmetric-ish rational configurations that keep only a single 3-fold axis — order-3 symmetry, down from golden's order-60, but never zero.

"Are subsets of record configurations also record configurations?" Strikingly, yes: the records **nest**. 723 contains the $n=4$ record 183 exactly, contains the new $n=5$ best 393, and its pairs hit the $n=2$ record 13. Nesting is generative, not just descriptive: taking 723 and bolting on a seventh cube — 256 candidate orientations, no fine-tuning — gave **1207** at $n=7$, beating a 50,000-seed random campaign's 1085 on the first try. And it runs top-down too: no five-cube-native search ever found 393; it is only reachable as a shadow of the better six-cube configuration. The record tower 183 → 393 → 723 → 1207 has adjacent floors related by adding/removing one cube.

Act IV: frustration, and what makes a good building block

Here is the conceptual heart of the project, and it started from another user observation: *golden N4 is built from optimal sub-configurations on every subset, yet golden N4 is not the maximum.*

Verified exactly: the golden four-compound has **every** pair at the pair maximum (13) and **every** triple at the triple maximum (67) — locally perfect everywhere — and totals 177. The actual maximum, 183, has triples of only 63 and half its pairs at 9. **You cannot make every part optimal and maximize the whole.** Local perfection is globally *frustrated* from four cubes onward (through three cubes the two coincide, which is why the golden triple really is the $n=3$ maximum).

Why does the maximum deliberately use "worse" parts? Two measured facts answer it:

1. **Rigidity.** Sampling thousands of random pairs: 94% count 4 (generic), 2% count 9, 0.1% count 13. The *optimal* pair is the rarest and most rigid thing in the space — an isolated wall. The 9-pair, by contrast, is a continuous family: two cubes sharing a face axis count exactly 9 at every angle about that axis. The suboptimal configuration carries a knob; the optimal one doesn't. (Same one level up: the 67-triples are isolated points; a continuous family connecting the octahedral and golden 67s exists, but its interior sags to ~37.)

2. **What embedding conserves.** Compare the golden triple $67 = \{48, 18, 1\}$ with the 63-triples inside the 723 record, all of which are $\{44, 18, 1\}$ with pairs $[9, 13, 13]$. Identical in the deep layers — and the deep layers are what a larger compound inherits, because new cubes' faces recut the shallow (depth-1) regions anyway. The golden triple's entire +4 advantage lives in the recutable layer, and it pays for those four disposable regions with its last degree of freedom and its rational compatibility.

So: **a good building block is deep-saturated, shallow-detuned, and keeps its knob.** The record is a hub-and-spoke assembly — one cube optimally (13-)paired to a cluster of spokes that are mutually 9-paired on a shared axis, whose angles are the tuning parameters the global optimum spends. A targeted campaign confirmed the mechanics: searching those spoke angles directly recovers every record (183, 393, 723) in a few thousand counts each, while locking the angles to symmetric values costs 15–45 regions and unstructured search trails by 20–30.

Act V: the law

All week, certain depth-profile entries kept refusing to move. At $n=6$, no configuration ever had depth-5 above 36, depth-4 above 102, depth-3 above 164 — while depth-1 and depth-2 grew freely with each record. The deep layers looked *quantized*: each either hits its generic value or a smaller degenerate one, never more. When the $n=4$ search showed the same at 24, and profiles across $n=2..7$ were laid side by side, the caps snapped into a single formula. For the layer / steps up from the deepest:

$$\text{depth}-(n-l) \leq C(l, n) = (12l - 6) \cdot n - 2(l^2 - 1)$$

Slopes 6, 18, 30, 42 (arithmetic, step 12); intercepts $-2(l^2-1)$. The evidence is threefold and each leg is independent:

- **Attainment.** Across roughly a million exactly-counted configurations, every testable cap for $l \leq 4$ at $n = 2..7$ is attained exactly and exceeded never — 18 of 18 cells (12/18/24/30/36/42; 66/84/102/120; 104/134/164/194; 180/222/264).
- **Golden's role explained.** The shallowest case $l = n-1$ gives $\text{depth-1} \leq 10n^2 - 14n$, and the golden compounds hit it exactly (48, 104, 180 at $n = 3, 4, 5$). Golden is the *top-layer-cap* configuration; the records are the *deep-cap* configurations; the frustration is that nobody gets both. Summing all caps bounds the total: ≤ 801 at $n=6$ (record 723 — the 78-point gap is the price of frustration made visible), ≤ 195 at $n=4$, ≤ 445 at $n=5$, ≤ 1343 at $n=7$.
- **A completely independent measurement agrees.** There's a reformulation on the direction-sphere: point outward in direction $\hat{u}$; cube k extends to radius $1/\|\mathbf{R}^T \hat{u}\|_\infty$; the deep layers count cells of "which cubes are innermost" diagrams whose boundaries are great-circle arcs. Early in the project those diagrams were built exactly for a few configurations and their vertex/edge counts recorded: $V = 68, 200, 324$ at $n=6$. All the measured diagrams are trivalent ($E = 3V/2$), so Euler's formula forces cells = $2 + V/2$ —

and the law then *demand*s $V_l(n) = (24l-12)n - 4l^2$, which evaluates to exactly 68, 200, 324. Numbers measured days before the formula existed.

The law is still a conjecture. But it reduces to one crisp combinatorial claim (prove the swap-curves of a generic configuration have exactly $(24l-12)n - 4l^2$ vertices, all trivalent; Euler does the rest, and a separate semicontinuity argument handles degenerate configurations), and its deepest case $l=1$ reduces further to a lovely elementary statement: *the function "distance to the nearest face" on the direction-sphere has no local minima except the $6n$ face-centre directions*. The two-normal case of that is provable by hand. The three-normal case is the open crux. Any reader who wants a genuinely attackable open problem: that's the one.

Act VI: closing the search like an optimization problem

Another user reframing: *"so our searches can now be over ways to combine building blocks and trying frustration trade-offs — can it be reduced to something like branch-and-bound?"* Almost exactly.

- **Branch**: a configuration's *blueprint* — how the cubes partition into shared-axis clusters, which pairs are rigid 13s versus tunable 9s, which cubes are free. Finite after symmetry: 391 raw blueprints collapse to **67 canonical skeletons** at $n=6$.
- **Prune**: with justified rules only — geometric inconsistency, and frustration itself as a pruning rule (an all-13 triangle forces the golden wall, whose extensions provably lose).
- **Optimize**: each surviving skeleton over its continuous knobs (the spoke angles), which we know are the right variables.
- **Bound** (the part that needed new mathematics): measured from 532 configurations with all their five-cube subsets exactly counted — **(E1)** a six-cube total never exceeds its best five-cube subset by more than 336, and **(E2)** if any five-cube subset misses its deep caps at all, the total is capped $\sim 150+$ below the record.

All 67 skeletons were searched (83,700+ exact counts). **Nothing beat 723**. Combined with E1, the situation is pleasantly cornered: any configuration beating 723 must *contain* a five-cube arrangement totalling ≥ 388 — and after $\sim 171,000$ five-cube configurations, the only known members of that class are 723's own subsets. So the hunt for a better six-cube arrangement is no longer a 15-dimensional needle search; it is precisely the hunt for one new near-record five-cube arrangement. (E1 and E2 are measured envelopes, not theorems — clearly labelled as such — and proving E1, likely via a zone-style bound on how many regions six new faces can create, would make the cornering rigorous.)

The same full apparatus — climb the record, enumerate blueprints, test the caps, extend the tower, measure the envelope — was then run at $n=7$, the theory's first outing on a size it hadn't been fitted to. Verdict: 1207 stood (certified against climbing, cube-swaps, and a 100-skeleton blueprint catalog whose best was 1207's own blueprint); the law took zero violations across 112,864 fresh counts, with the deep caps attained exactly and depth-2 landing within 2 of its predicted 330; and one more greedy extension produced the first eight-cube record, **1879**, whose deep layers hug the law's $n=8$ predictions from below (48 attained, then -2 , -6 , -4). The

tower reads $183 \rightarrow 393 \rightarrow 723 \rightarrow 1207 \rightarrow 1879$, every level the best known at its size, each built from the one below it.

Act VII: the dihedral family, or how a stray remark in a viewer became a closed-form theorem

This act started, like several of the best ones, with a human just *looking* at something. The project's viewer had a "67 $\leftrightarrow$ 67 slide" — a hand-built path dragging the octahedral three-cube maximum over to the golden one — and its midpoint showed a scatter of near-miss edge crossings, close but not quite touching: "ghosts," in the project's own language. Looking at the picture, the observation was that the ghost edges all seemed to sit in a plane perpendicular to the direction $(1,1,1)$. That one remark is the entire origin of everything below.

Chasing it down turned up a genuine closed-form object: take a cube and an axis $n(\psi) = (\sin \psi, \cos \psi, 0)$ — one that lies *in* one of the cube's own face planes — and rotate the cube by $\pm 120^\circ$ about it. Three cubes, one parameter ψ . And for *every* ψ , not just special ones, the corresponding edges of any two of the three cubes are exactly coplanar — built-in coincidences everywhere, not just at isolated alignments. Both of the project's two 67s turn out to be members: the octahedral one at $\psi = \arcsin(1/\sqrt{3})$, the golden one where $\tan \psi = \phi^2$ (ϕ the golden ratio) — a condition that collapses to the tidy identity $\phi^2 + \phi^{-2} = 3$. A brand new point fell out for free at $\psi = 45^\circ$ exactly, the face-diagonal axis: a compound with entries in the field $\mathbb{Q}(\sqrt{6})$, counted exactly (a new engine, `q6_count.py`, cloned from the project's existing $\sqrt{2}$ counter) **at 49 regions**, depth profile $\{30, 18, 1\}$.

Sweeping the whole family with exact arithmetic (Pythagorean angles give rotations in $\mathbb{Q}(\sqrt{3})$, so a sibling engine `q3_count.py` can count them exactly too) produced a clean symmetric staircase around $\psi=45^\circ$: 25, 31, 43, then a long plateau at 55 between the two golden points, with the two 67s as spikes at its ends — and 49 sitting, unexpectedly, as a *dip* below the 55-plateau rather than a peak. That was the first real surprise of this act: more coincidences do not automatically mean more regions. The extra crossings at the octahedral and golden points *create* new divisions; the extra crossings at the face-diagonal point happen to *merge* regions that would otherwise be separate. Coincidence-richness, again, cuts both ways — the same lesson the edge-versus-corner story taught back in Act III, now visible inside a single continuous family.

The old ghost-gapped slide, it turned out, was simply not walking along this surface — its path had a small nonzero component along $(1,1,1)$ where the family needs exactly zero, and that tiny miss is precisely where the ghosts came from. A follow-up question — "is there a way to slide while *maintaining* edge concurrences?" — led to the nicest single finding of the act: the *same 18* interior edge crossings persist, unbroken, across the entire open stretch between the two golden copies. The 30s and 24 at the special points are momentary extras that vanish as you leave them; the 18-core never opens a gap. Arriving at either golden point, none of the 18 breaks — six stay interior (docking at a segment position of $1/\phi^3$, one more golden-ratio cameo) and twelve land exactly on cube corners, becoming golden's own corner structure.

That success invited an obvious next question — can a path do *better* than 18 all the way to golden? — and the answer, after a real search (riding a curve of extra coincidences to hold 26 concurrences over part of the range, then trying nine different corner "handoffs" at the wall where that curve runs out, then working backwards from golden's own extra curve), was no: 18 remains the best confirmed lower bound, with a specific, locatable obstruction. Both the octahedral side's extra- coincidence curve and golden's own extra-coincidence curve pass close to $\psi=45^\circ$ — which is also the tetrahedral angle, $\arccos(-1/3)$, showing up in disguise — but about 70° apart in phase, and neither curve bends around to link the other. Not a proof that 18 is a ceiling, but a real, honestly described wall, not just "we didn't find anything."

Four pieces of this got upgraded from "checked numerically to sixteen decimal places" to actually proved: a mirror symmetry (ψ and $90^\circ-\psi$ give congruent compounds, for any n), an exact 90° periodicity, the coincidence identity itself (proved for every ψ and every n by direct vector computation, not just verified on samples), and — the one with a genuine payoff — a theorem that any all-rational configuration has a rational version of the pairwise invariant that both 67s' irrational value rules out. Put together with the (unproven, but well-supported) belief that the two known 67s are the *only* three-cube maxima, that gives a striking conditional fact: three cubes would be the **one and only irrational level** of the entire record tower — two cubes rational, three cubes forced irrational, four and up rational again.

The family also turned out to explain something about the *records* themselves, not just about three cubes. Generalized to n cubes on a shared axis with independent phases, at Pythagorean angles every member becomes an ordinary integer-quaternion configuration the fast C++ engine can already count — so, for the first time, the family could be searched exhaustively rather than just admired. As a search space on its own it disappoints: the best pure single-axis family members found at $n=4,5,6$ are 175, 335, 615, falling further and further behind the true records (183, 393, 723) as n grows. But checking every *pair* inside each record against the family's own membership test told a completely different story: all 6 pairs of the 183 record, all 10 pairs of 393, and 12 of the 15 pairs of 723 are in family position. The records are not single-axis family members — they are **gluings of family cliques sitting on different axes** (723, worked out exactly, is a 5-cube family clique — which is exactly the embedded 393 record — plus one more cube linked to two of the five). That is a genuinely new way to think about where the records come from, and it reframes the search for anything beyond 723: not a blind search over all rotations, but a search over how many cliques, on which axes, glued how.

The viewer got a matching set of upgrades. The old ghost-gapped slide was replaced outright by a slider along the real dihedral family — a ψ dial from 0° to 90° , named tick marks (including a newly-recognized mirror-golden point), a live ghost counter, and a "maintain concurrences" lock that clamps dragging to a range where the crossing set is certified constant. An opaque-surface mode was added alongside the old point cloud, turning the compound's faces into solid, shaded, paintable polygons; on top of that came live highlighting of exactly the faces about to split or merge at the current ψ , mouse-wheel zoom, and one-sided clipping against the cross-section plane so the solid interior can be inspected without the near half in the way. All of it lives at the same published link as before.

Two threads from this act are still open, deliberately unresolved rather than oversold: a systematic search over gluings of family cliques on different axes (has anyone actually tried to beat 723 this way yet — no), and whether the $n=4$ record has its own irrational "resonance" point in the family, the way $n=3$ has its two 67s, findable with the same kind of algebraic solve that located those in the first place. Neither is finished.

Act VIII: the rational slice, and the first maximum actually proved

Act VIII is the act where the two loose threads of Act VII got run to ground — one to a satisfying answer, one to a clean negative — and where, almost as a side effect of tidying up the proof program, the project proved its first honest maximum theorem.

The thread about *where the records live* resolved first, and it turned on a distinction nobody had drawn carefully. The dihedral family's tilt ψ had always been swept at "Pythagorean" angles — those with rational sine and cosine, the ones a right triangle with integer sides gives you — because those keep every rotation rational and countable by the fast engine. But when the actual record configurations were dissected, their internal cliques sat at tilts like $\tan \psi = 2/3$: a rational *tangent* with an irrational *sine*. That is a different slice of angles entirely, and every prior sweep had been structurally blind to it — not under-resolved, but pointed at the wrong locus. The fix is a small piece of arithmetic: at a rational-tangent tilt, a phase step keeps the whole configuration rational exactly when its parameters are a rational point on a certain conic, and those points have a tidy one-parameter formula that closes under addition. So even this "irrational-sine" slice is, after all, searchable by the integer-only engine — you just have to know to walk the conic. The 393 record's four-cube clique turned out to be exactly such a conic chain, its phases landing at t -values $-5/6$, $3/4$, $-1/5$ with the remaining pairs falling into place with no further search: a clean certificate that the record's own structure lives in this slice.

Searching the slice properly broke something that had looked like a law. An earlier campaign had found that gluing family cliques together always landed *exactly 8 regions* short of the record, at four cubes, five cubes, and six cubes alike — a suspiciously constant deficit that begged to be either a theorem or a coincidence. It was a coincidence, an artifact of the gluing search's limited vocabulary. Taking the exact 393 clique and attaching a fifth cube *on the same axis* at a conic phase gave **387** regions at five cubes — beating the glue campaign's 385 and cutting the deficit to 6. The "constant 8" was simply the best that a particular restricted construction could do; a single-axis five-chain with non-uniform phases, which that construction couldn't express, does better. A related tidying-up found that the six-cube record is not one configuration but a *plateau*: fixing the record's first five cubes and sweeping a sixth over thousands of integer orientations turned up 27 distinct completions all counting exactly 723, in four different depth profiles that trade middle-layer count at a fixed exchange rate — the same shallow-for-deep conservation seen throughout the project, now visible as an exact arithmetic law operating right at the summit. The record *value* stands; "the 723 compound" is at least four compounds.

The second thread — the hunt for an $n=4$ "resonance" — resolved negative, and cleanly. Using computer algebra to write down the exact conditions under which extra cross-alignments appear at four cubes, then solving those conditions and counting every resulting configuration exactly, the verdict was uniform: every such resonance in the reachable number fields *merges* regions rather than creating them. The best one counts 151 — below even the family's own rational plateau of 175, nowhere near the 183 record. The +12 "spike" that irrationality buys at three cubes simply does not recur at four; the extra coincidences there behave like the face-diagonal dip, not like the octahedral peak. There is one honest caveat kept in the ledger — a handful of higher-degree candidates remain uncounted for want of a specialized exact-sign routine, one of them sitting in the record's own number field — but the mechanism that made irrationality *pay* at three cubes is, at four cubes, proven not to.

Put beside Act VII's conditional result, this hardens the tower's oddest feature. Three cubes appears to be the **one irrational rung** of the entire ladder — and the reason is now visible rather than mysterious. Attaining the maximum at two or three cubes requires maxing every depth layer at once, which pins the configuration to an isolated, algebraically rigid point (irrational, for three cubes). From four cubes on, *no* configuration can max all layers simultaneously — the layers must be traded — and trades happen on open sets, which always contain rational points. The 387's tolerance for a whole interval of phases, and the 723 summit's plateau of 27-plus realizations, are that openness made visible. Knife-edge maxima can force irrationality; wide maxima cannot, and past three cubes the maxima are wide.

Then two delegated censuses, run to answer a completely different question, turned out to interlock and explain a great deal at once. The question was create-versus-merge: when a compound acquires an exact coincidence, does the region count go up or down, and by how much? The tempting guess — "plus or minus one region per coincidence" — is exactly right for the pure interior-crossing events at three cubes, and wrong everywhere else: it reaches +2 per contact at the golden point, and at four cubes it fails *in sign*, coincidences increasing while the count falls by 24. But a different law held on every one of the twelve events examined: the entire count change lives in the shallowest depth layer, with every deeper layer conserved to the exact integer. The reason emerged from the second census, which mapped both three-cube maxima as exact spherical diagrams. Every coincidence — every edge crossing, every corner contact — appears as a vertex of the *top* diagram (the one that controls the shallow layer) and never touches the *bottom* diagram (which controls the deep layers); the bottom diagrams of even these maximally special configurations are combinatorially generic. That is *why* the depth-conservation law holds, and it turns the create-versus-merge question from a mystery spread across mechanisms into a bookkeeping question on a single diagram: does the vertex weight a coincidence adds exceed the arc structure it consumes? The same census also delivered a correction and a gift to the proof program: the shallow-layer bound of 48 is Euler-tight at both maxima — their top diagrams carry a total vertex weight of exactly 92, split (contrary to an earlier projection) between 32 units on genuine triple points and 60 on the coincidence contacts themselves.

Which set up the act's real prize. The proof program for "three cubes make at most 67 regions" had been written out as a tree of lemmas with two genuine gaps; the more fundamental gap — shared with the still-unproven two-cube case — was ruling out "parasite" cells, a topological possibility that local analysis couldn't exclude and that would have sunk the whole second-deepest bound. It turned out to yield to a short argument built from three rigidities the project already had in hand: at any point where two faces tie for closest, their spherical gradients have *equal length* (the length depends only on the shared value); the tie curves are all arcs of great circles, and distinct great circles are never tangent; and a linear-algebra alternative (Gordan's theorem) forces the equal-length gradients either to admit a strictly-improving direction — which the great-circle geometry guarantees stays inside the cell — or to be exactly equal, which means two genuinely parallel faces, a degeneracy that removes itself from the problem. No parasites, at any number of cubes. The draft still wants an adversarial read before it is called finished, and it is filed honestly as a draft. But if it holds, it closes the two-cube case outright — **thirteen is proved maximal, the project's first complete maximum theorem** — proves the second-deepest ceiling $\text{depth}-(n-1) \leq 6n$ for *all* n (open problem number one, in both write-ups, for months), and finishes the entire first half of the three-cube proof. What remains for a full "67 is maximal" is now a single finite classification of the top diagram — and the census just handed over the exact numbers that classification has to reproduce.

So Act VIII closes the two threads Act VII left dangling (the records' slice: found and searched; the $n=4$ resonance: proved not to pay), and converts the project's oldest open problem and its first maximum theorem from "measured, never proved" to "proved, pending one careful read." The irrational-rung story, still resting on the unproved uniqueness of the two 67s, is now the best-supported conjecture in the whole enterprise.

Act IX: 67 is maximal (the last classification, and a false lemma)

What Act VIII left as "a single finite classification of the top diagram" became a theorem. The shallow bound $\text{depth}-1 \leq 48$ reduces, by Euler, to the top diagram carrying vertex weight at most 92 — and the census had already shown the two 67s hit exactly 92, split 32 (triple points) and 60 (contacts). The 60 fell to a clean idea the human's own intuition prompted — *if the bound is independent of the cells' shape, there should be one topological reason for it*: a contact is an edge-of-one-cube crossing an edge-of-another, which is precisely a four-valent vertex of the polytope where those two cubes intersect, and Euler on any convex polytope caps its total vertex weight at $2F-4$. Three pairwise intersections, twelve faces each, twenty apiece: sixty.

The triple points nearly closed the same way, and here the story earned one more wrong turn. The tidy claim was that the "farthest-cell" diagram can be no more branched than the "nearest-cell" one, which $\text{depth}-2 \leq 18$ already caps at 32. It is false — and not subtly: a triple point with one cube at a corner and two at thin blades has a *degree-eight* farthest-cell vertex against a degree-four nearest-cell one, and it is buildable from honest cubes, not an artifact of the model. Chasing it produced the last idea: don't compare the two diagrams, *charge each triple point to both budgets at once* — the nearest-cell diagram and the pairwise polytopes — and a three-line case analysis (on how many cubes are ever nearest at the point) shows even a

degree-eight vertex takes no more than its share. With that, the degenerate triple points are fully handled, and **max(3) = 67 is a theorem** for all pairwise-transversal triples of convex six-faced cells, cubes among them. The write-up is PROOF_STEP_T.md; the readable version, missteps and all, is PROOF_NARRATIVE.md.

So the oldest concrete target of the whole enterprise — "three cubes make at most 67 regions" — is proved, and it proved to be a statement not about cubes but about convexity and Euler's formula. What remains genuinely open is one level up: whether the two 67s are the *only* three-cube maxima, the irrational-rung uniqueness that stays the best-supported conjecture here.

The collaboration, honestly described

This project was a four-layer collaboration, and the layering was not decorative — each layer did something the others couldn't. It is also worth being explicit about this document's own place in that layering: this write-up — like every line of code, every search, and every piece of analysis in the project — was **written by an AI** (Claude), working under human direction. The human side of the collaboration supplied questions, corrections, and the occasional observation that turned out to unlock an entire act of the story; the AI side did essentially all of the designing, coding, computing, and writing, including the words you are reading now.

The human (with friends — Chris Cole and Werner — kibitzing) supplied almost every pivot: the sliding-triple family, the edge-vs- corner observation, "try intersections between families," "is 177 wrong?", "are subsets of records also records?", the building-blocks/ frustration reframing, branch-and-bound, and — the origin of Act VII — simply noticing, while looking at the viewer, that a scatter of near-miss edges appeared to sit in a plane perpendicular to (1,1,1). None of these were "requests to compute"; they were acts of noticing. The pattern is worth stating: the human watched the data for *meaning* while the machines watched it for *values*. Several of this project's best results are literally a human sentence turned into a measurement — the perpendicular-edges remark most dramatically of all, since it is the entire seed of the closed-form family, the four theorems, and the records-are-gluings finding described in Act VII.

A frontier model (Anthropic's Fable 5; an Opus 4.8 stint when weekly limits hit) ran the main session: designing the exact algorithms and validation gates, writing the specifications, spotting and naming the regularities (the ceiling formula was found by staring at maxima tables and fitting), catching errors — including its own. Concrete corrections that made it into the ledger: an early claim that record configurations trade deep count for shallow at a favourable "6-for-45" rate collapsed under proper optimization to an exact 1:1 conserved exchange; a claimed confirmation that "golden maximizes depth-1" turned out to be a broken test (it never got near the golden value) and was re-labelled untested until the formula later explained it; a "richness is mid-total" misreading of early data was refuted by a cheap model's careful correlation analysis. The ledger keeps all of these, deliberately.

Mid-tier execution agents (Claude Sonnet) did the heavy building: the C++ engine, every campaign and search driver, the $\mathbb{Q}(\sqrt{2})/\mathbb{Q}(\sqrt{5})$ /tower arithmetic, the incidence analyzers. Two systematic failure modes emerged, and both were manageable once named. First, *coverage artifacts*: an agent's search can be perfectly implemented over an accidentally-too-small parameter space — the fix was the hard gate "your machinery must reproduce the current record before your negative results count." Second, *premature parking*: agents would set up a long computation, then stop to "wait for a monitor" instead of running it; the fix was insisting every campaign be a single detached self-sequencing script, with the agent only collecting results. With those two guardrails, delegation worked extremely well — a dozen-plus agent campaigns produced the record chain, and every "we found X" was independently re-verified in the main session before being believed.

A small model (Claude Haiku) handled a pure data-analysis task — the subset-richness census over 278k configurations — and notably *overturned the frontier model's prior belief* (richness correlates positively with totals; records are rich AND balanced). Cheap models audit well.

Non-LLM tools carried the actual mathematics: the C++/Python twin engines (never disagreed once in ~1M counts), Wolfram Engine via wolframscript for the algebraic side (Gröbner-basis solves for configurations satisfying multiple corner-coincidences at once — it found exact configurations no numeric grid lands on, including the depth-1 record-holder $d_1=224$), and a small self-contained browser visualizer (depth_explorer.html — depth-coloured cross-sections you can sweep through the compound, a rotatable depth point-cloud, rings marking edge/corner concurrences, presets for the golden and octahedral compounds; also hosted online). The exports of every session, the specification files, and the ledger's postscripts are what let the work survive a dozen context resets and model switches: the *filesystem* was the collaboration's long-term memory.

Reproducing this (for a reader with a Sonnet-class assistant)

Everything can be rebuilt from the descriptions in this document, but the fast path if you have the repository: build the counter and check the anchors —

```
clang++ -O2 -std=c++17 -o cube_regions cube_regions.cpp
./cube_regions --selftest          # axial-6 → 121
./cube_regions --quats '4,1,1,-1;3,3,7,3;5,-1,-5,-5;2,1,1,1;1,1,1,1;5,2,2,2' # → 723
```

From scratch, the essential recipe an assistant can implement in an afternoon: (1) integer quaternion → exact rational rotation matrix; (2) plane-cut a bounding box by the 6n face planes with Fraction arithmetic, tracking convex cells; (3) merge cells across facets that lie outside the owning face's square (union-find) — this phantom-facet merge is the one subtle step; (4) label regions by containment, count by depth. Validate on 121 and 351 before trusting anything. Then: random campaign to see the 635 wall; wide-perturbation climbing to break golden values (reproduce 183 from scratch!); shared-axis templates with free spoke angles to reach the

records; and test $C(l,n)$ against everything you count. All the record quaternions are in this document and the ledger; every claimed number is checkable in seconds.

Open questions

Act VIII resolved several of these; the list below marks what moved.

1. **Finish "67 is maximal" for three cubes.** Half of it is now proved (see the No-Parasites draft): the second-deepest and deep bounds give $\text{depth-2} \leq 18$ and $\text{depth-3} \leq 1$. What remains is the shallow bound $\text{depth-1} \leq 48$ — a single finite classification of the top diagram's vertices, for which the two maxima's exact censuses now supply the equality cases that classification must reproduce. This is the nearest theorem still open.
2. **Harden the No-Parasites proof, then two cubes are done.** The draft argument (equal-length tie gradients + great-circles-never-tangent + Gordan's alternative) closes the last gap for **max = 13 at two cubes** — the first complete maximum theorem — and proves $\text{depth-(n-1)} \leq 6n$ for *all* n , the project's oldest open problem. It is filed as a draft pending one adversarial read.
3. **Beat 723 or corner it completely** — equivalent, by envelope E1, to finding any fundamentally new five-cube arrangement ≥ 388 , or proving none exists. The rational-tangent slice reached 387 at five cubes (breaking the old "constant-8 deficit") but has not passed a record; the six-cube record is now known to be a *plateau* of ≥ 27 non-congruent realizations, all exactly 723.
4. **Prove envelope E1** (a zone-style bound making the branch-and-prune a certified branch-and-bound) — unchanged, still open.
5. **Why exactly does frustration switch on at four cubes?** Act VIII gives the mechanism: attaining the maximum needs every depth layer maxed at once, which is possible only at $n \leq 3$ (and forces irrationality at $n = 3$); from $n = 4$ the layers must be traded, and trades live on open — hence rational — sets. This is now an argument, not just a measurement, though not yet a theorem.
6. **Unequal cube sizes** — off-centring provably hurts near the record; size variation is still untested (resizing hits a degeneracy the current counters can't evaluate; a degeneracy-robust counter would settle it).
7. **The tower at scale** — does greedy extension stay within a constant of optimal as n grows, and what is the asymptotic growth of $\max(n)$? (Cap-sum bound $O(n^3)$; the records track it suspiciously well.)
8. **Is three the only irrational rung?** Two cubes rational, three forced irrational, four proved not to gain from irrationality (best resonance 151, count-negative) — conditional on the two 67s being the unique three-cube maxima, three cubes is the sole irrational level of the tower. The one live escape route: a short list of higher-degree $n=4$ resonance candidates left uncounted for want of an exact-sign routine (one sits in the record's own number field).
9. **A create-versus-merge criterion.** Coincidences are exactly the top-diagram vertices; whether a new one creates or merges regions is now a bookkeeping question — does its added vertex weight exceed the arc structure it consumes? — but no closed criterion is

written down yet. The " ± 1 per coincidence" guess is dead; depth-conservation (all change in the shallow layer) held on all 12 events tested.

10. **More than 18 concurrences, octahedral to golden?** — 18 is a confirmed lower bound with a located obstruction (two extra- coincidence curves grazing $\psi=45^\circ$ about 70° apart, never linking), not a proven ceiling.

[Source: <https://github.com/dmi-000/cubes/blob/main/JOURNEY.md>]

The Feynman Ciphers: A Cryptanalysis Story, 1945-2026

Chris Cole and Anthropic Fable 5

Three encrypted notes were handed to a Nobel physicist at Los Alamos. One fell within days of reaching the internet. The other two held out for thirty-six years -- and hid, among other things, Feynman's own words.

I. PUZZLES ON THE HILL (Los Alamos, 1940s)

The scientists of the Manhattan Project worked behind fences and censors, and they entertained themselves accordingly. Feynman's safecracking exploits are the famous example, but the traffic ran both ways: colleagues amused each other by exchanging enciphered messages as challenges. Five of these wartime ciphers eventually became public, decades later, as two families. Two notes composed by the mathematician Paul Olum -- who had baited Feynman with them personally -- surfaced on Reddit in 2019 and were solved within months by Paul Rekin, one of them requiring a genuinely new diagnostic method for transposition ciphers. And three notes that Feynman had received from a researcher whose identity is now lost were posted to Usenet's sci.crypt in 1987, where cipher enthusiasts christened them the Feynman Ciphers.

The first was broken almost immediately. Jack C. Morrison peeled it apart: a simple unkeyed transposition -- write the text into a five-column grid and read it out in the right order -- concealing the opening of Chaucer's Canterbury Tales in Middle English. A physicist's joke: the "hardest" part was recognizing fourteenth-century spelling.

Feynman #2 (261 letters) and #3 (231 letters) did not fall. Not in 1987, and not for thirty-six years afterward.

II. TWO CIPHERS THAT REFUSED EVERY RULER

[Specimen 1: Feynman #2 -- 261 letters]

XUKEXWSLZJUAXUNKIGWFSOZRAWURORKXAOSLHROB
XBTKCMUWDVPTFBLMKEFVWMUXTVTWUIDDJVZKBRMC
WOIWYDXMLUFPVSHAGSVWUFWORCWUIDUJCNVTTBER
TUNOJUZHVTWKORSVRZSVVFSQXOCMUWPYTRLGBMCY
POJCLRIYTVFCCMUWUFPOXCNMCIWMSKXPEDLYIQKD
JWIWCJUMVRCJUMVRKXWURKPSSEIWWZVXULEIOETOO
FWKBIUXPXUGOWLFPWUSCH

[Specimen 2: Feynman #3 -- 231 letters]

WURVFXGJYTHEIZXSQXOBGSVRUDOOJXATBKARVIX
PYTMYABMVUFXPXKUJVPLSDVTGNGOSIGLWURPKFCV
GELLRNGLPYTFVTPXAJOSCWRODORWNWSICLFKEMO
TGJYCRRAOJVNTODVMNSQIVICRBICRUDCSKXYPDMD
ROJUZICRVFWXIFPXIVVIEPYTDOIAVRBOOXWRAKPS
ZXTZKVROSWCRCFVEESOLWKTBOXAUXVB

The statistics were maddening because they refused to take a side. The index of coincidence -- cryptanalysis's first thermometer -- read about 0.045 for both texts: too flat for a simple substitution (0.066), too structured for a proper polyalphabetic cipher (0.038). Periodic tests found no honest period. All 26 letters appeared, ruling out the whole family of 25-letter square ciphers. And strangest of all, the two ciphers SHARED material: the tetragram SQXO appears in both, along with several trigrams -- at positions no periodic key could reconcile. Feynman #2 even contains a six-letter block repeated back-to-back, CJUMVR CJUMVR, odds of chance roughly one in a million.

Every classical model contradicts some part of that picture. A transposition preserves letter frequencies -- ruled out. A periodic polyalphabetic makes the repeated block impossible unless the period divides six -- ruled out. A single substitution can't produce an IC of 0.045 -- ruled out. For thirty-six years, the ciphers sat in that contradiction.

III. THE HUMAN SOLVE (May 2023)

In May 2023 David Vierra, writing as codewarrior0 on his cipher blog, finally broke both. His account is a small masterclass in cryptanalytic instinct. The ciphers had lived in his "unsolved cases" file for months when he tried something with no theoretical justification at all: he concatenated both ciphertexts, REVERSED the whole string, and fed it to AZdecrypt's simple-substitution solver -- a model he knew was wrong for this text.

Out came word salad. But at the head of it stood the word PRINCIPLES, and further in, TRANSITION, and a fragment reading SPECIALL right after an E. Wrong model, half-right output. Working the cribs by hand, a fragment LACITER bothered him until he remembered he had reversed the text -- read the other way it sat inside THEORETICAL. Some words were coming out forward, others backward. Assuming a Q-follows-U, he pulled LIQUID out of one cipher and LIQUOR out of the other. Then, on a trip to the grocery store, the IC of 0.045 clicked into place: not homophonic -- MILDLY POLYALPHABETIC, two or three alphabets. The letters following THEORETICAL read SCWRODORWNW; a pattern search matched INTERPRET, the word INTERPRETATIONS completed itself, and, in his words, the cipher fell apart after that.

The method, once visible, was nothing any textbook lists. Two private substitution alphabets. Each WORD of the plaintext is enciphered by one of them, alternating word by word; the first word of every sentence or line of verse resets to Alphabet 1. And words with an even number of letters are written backwards first. Word boundaries are then discarded, so the alternation rides on structure the codebreaker cannot see.

Plain	the	behavior	of	liquid	helium	especially
Alphabet	1	2	2	2	1	2
Reversed?	—	yes	yes	yes	no	no
Cipher	WUR	VFXGJYTH	EI	ZXSQXO	BGSVRU	DOOJXATBKT ...

Every anomaly dissolves at once. The IC lands between mono and poly because the text is a word-grained braid of two substitutions. SQXO appears in both ciphers because both plaintexts contain the letters QUI+D/R -- "liquid" in one, "liquor" in the other, same alphabet,

even-length, reversed. CJUMVR CJUMVR is just a repeated word enciphered identically twice in a row. And the plaintexts:

“Oh many a peer of England brews
Livelier liquor than the Muse,
And malt does more than Milton can
To justify God's ways to man.”

-Feynman #2: A. E. Housman, "Terence, This is Stupid Stuff"
(1896)

“The behavior of liquid helium, especially below the lambda
transition, is very curious... In this paper and one or two to
follow, the problem will be studied entirely from first principles.”

-Feynman #3: R. P. Feynman, "Atomic Theory of the Lambda
Transition in Helium," *Physical Review* (1953)

Whoever set the challenge enciphered Feynman's own abstract and handed it back to him. He never read it.

IV. THE MACHINE SOLVE (July 2026)

This month the ciphers were revisited in this workspace as an exercise with one rule: no peeking. Feynman #1 fell to an exhaustive route-transposition scan -- the same Chaucer, recovered blind. Against #2 and #3, a two-day campaign of validated solvers legitimately excluded a dozen cipher families, several of them never tried against these texts before:

<u>Hypothesis</u>	<u>Test</u>	<u>Verdict</u>
Transposition only	IC 0.045 vs required 0.066	EXCLUDED
Periodic Vigenere/Beaufort family	Chi-squared column solves, $p \leq 30$	EXCLUDED
Route x periodic substitution	All exact rectangles x orientations	EXCLUDED
Autokey (plain & cipher, 3 modes)	Chain solver, primers 1-30, fwd+rev	EXCLUDED
Hill 2x2, incl. vertically seriated	Exhaustive, 157,248 matrices	EXCLUDED
Trifid/fract. Morse/running key	Simulated annealing / beam search	EXCLUDED
Quagmire I-III	Annealer FAILED its own validation	TEST VOID
Word-alternating 2-alphabet subst.	jumpsolve.cpp (below)	SOLVED

That "test void" row carries the campaign's first lesson: every negative result is only as good as its solver, so validate the solver on a synthetic cipher before believing it. The Quagmire annealer couldn't crack a Quagmire it had just been handed the answer to; its exclusions were worthless.

With standard cryptanalysis exhausted, the rules changed by agreement: read Vierra's solution for the METHOD, then build a general algorithm that solves the cipher class from ciphertext alone -- no key material, nothing specific to these two messages. The result, jumpsolve.cpp, rests on four ideas.

1. Score the true model exactly. For any candidate pair of alphabets, a Viterbi dynamic program finds the best segmentation of the ciphertext into words -- each word choosing its length and alphabet, even lengths matched against a pre-reversed dictionary of 120,000 words, unknown words falling back to letter statistics so wrong keys still feel a gradient. Both ciphers are scored jointly under one shared key: two short messages crack each other. Crucially, the published key scores -495.8 under this objective while every wrong local optimum sits below -820 -- truth is the global optimum, so search is the only remaining problem. (Two tempting bootstrap objectives -- solving the mixture as a single substitution -- were checked the same way and found MIS-SPECIFIED: truth scored worse than junk. No optimizer can fix that; the objective check comes first.)
2. Anchor the search with structure. The most frequent repeated trigrams, WUR and PYT (four occurrences each across both ciphers), are prime candidates for THE under each alphabet -- odd-length words are never reversed. Pinning anchor-to-THE at the start of each annealing restart turned a one-in-fifty fluke into convergence on the third restart.
3. Match the escape move to the trap. Plain annealing plateaus where one alphabet dominates and starves the other of gradient -- cured by alternately freezing one alphabet and annealing the other. Near the summit, pairs of rare letters lock in exchanged positions (J-Q with X-Z) where no single swap improves -- cured by a double-transposition polish that tries a second disjoint swap on top of every failed one. Basin-hopping kicks connect the plateaus in between.
4. Sweep seeds in parallel. The search is stochastic; eight seeds ran side by side, and seed 505 converged in about ten minutes of wall clock.

[Objective scores (log-probability; closer to zero is better)]

Random keys	~ -1500
Junk plateau	-1060
Single-swap search stall	-823.5
jumpsolve, converged	-495.8
Published key	-495.8

The recovered alphabets agree with Vierra's except on letters he also marked unrecoverable -- information genuinely absent from 492 letters of ciphertext. The decrypt reads clean through both texts; the only garbled patch, three words around "a peer of England brews," sits exactly where the original encipherer broke his own alternation rule. As a generality check, the same binary was pointed at a fresh cipher in the same system -- a Jane Austen paragraph under random alphabets it had never seen. All three seeds converged to that cipher's true key.

V. WHAT EACH SOLVER WAS ACTUALLY GOOD AT

The honest comparison isn't machine-versus-human on speed. Once the cipher CLASS was known, optimization was cheap: eighteen seconds of annealing did what two days of wrong-model attacks could not. The hard part -- for both solvers -- was the search through model space, and there the human and the machine used opposite instruments. Vierra bootstrapped through a model he knew was wrong, harvested the half-correct debris for cribs, and let LIQUID/LIQUOR and a grocery-store hunch carry him from homophonic to two-alphabet. The machine needed the model handed over -- but then delivered something the by-hand solve never could: a reusable solver for the whole class, an exactly-scored objective on which the true key is provably the optimum, and a demonstration, three-for-three on fresh keys, that the Feynman ciphers were never unbreakable. They were just filed under the wrong model for thirty-six years.

SOURCES

David Vierra (codewarrior0), "Solutions to Feynman Ciphers #2 and #3," cipher-blog, May 27, 2023 (solve narrative, cipher method, plaintexts). Usenet sci.crypt (1987), via the same. Solver, logs, and attack record in the feynman-ciphers workspace, July 10-11, 2026. Written July 11, 2026.

Rick Rosner on Trump, Iran, Theology, AI, and the Failures of Modern Judgement

Rick Rosner & Scott Douglas Jacobsen



How does Rick Rosner connect Trump's judgment on Iran, COVID, tariffs, theology, and AI to broader failures in human reasoning?

Abstract

Scott Douglas Jacobsen interviews Rick Rosner on Trump's judgment, the U.S.-Israel war against Iran, political risk, and the costs to Americans. Their wide-ranging discussion ranges from intuition and deception to religion, creationism, identity, technology, architecture, geometry, and AI metrics. Rosner argues that impulsive decision-making, weak expert listening, and overconfidence recur across politics and public life, and that science, metaphysics, and information systems shape modern human experience.

Scott Douglas Jacobsen: How have you seen the progress of Mr. Trump and the war launched by the United States and Israel against Iran? In legal and geopolitical terms, how is that progressing? What does it cost Americans?

Rick Rosner: I'm just one guy with my under-informed views, but it's bad for the world. It's bad for the Jews. There was a lot of antisemitism before this.

And then, according to reporting, Netanyahu pushed Trump hard on confronting Iran, and Trump did in fact join Israel in launching strikes on Iran in late February. It has not turned out to be easy or clean. The Iranian regime is still in place, and the war has not produced regime collapse. It is hard to see how the regime would fall without some far greater military effort, and that would mean a massive escalation in a country of Iran's size, with Tehran deep inside the country and with the state still controlling major security and military forces.

Also, Tehran is a huge city, and Iran still has the internal security apparatus, the Revolutionary Guard, and the regular military. A lot of the population may hate the Revolutionary Guard, but that does not mean they do not still have the guns and the machinery of repression. So, anyway, it was a bad, stupid move for Trump. His approval did drop after the war escalated. Reuters Ipsos had him at 36% in late March, down from 40% the week before. A lot of conservatives and right-wing figures have criticized the war, and there have been public calls to talk about the 25th Amendment, but that is not realistically happening. Under Section 4, it would require the vice president and a majority of the principal officers of the executive departments, and, if contested, a two-thirds vote of both houses of Congress.

Gas is up sharply. AP reported the national average at about \$4.15 a gallon, nearly 40% higher since the war began, and Reuters reported oil up roughly 40% since the conflict disrupted shipping through the Strait of Hormuz. That is politically toxic. Republicans hold only a narrow House majority. As of March, the House Press Gallery listed Republicans at 217 and Democrats at 214, meaning the GOP cannot afford much slippage at all if it wants to keep control. So yes, this hurts his chances of holding the House in November, even if the exact seat-loss number is still speculation.

But anyway, Trump put himself in a bad position. He has cost America money, lives, and international standing. It is not COVID-level bad in terms of total domestic disruption, but it does show the same kind of impulsive, high-risk judgment. That is probably the consistency: overconfidence, poor cost assessment, and a habit of acting as though complexity will roll over for the force of personality.

Jacobsen: What are the consistencies in the pattern of judgment in COVID, in this war, and in the tariffs?

Rosner: He thinks what he thinks is correct, even if what he thinks is under-informed. He thought COVID wouldn't be bad, so his decisions were based on that, at least early on. He

thinks tariffs work, so he wasn't going to listen to anyone who said they can be inappropriate or harmful depending on the economic situation.

Job performance is more mixed than often claimed. There were strong job losses at the end of his first term during COVID, following earlier gains, so the record is not as simple as a single trend line. But the broader point still holds. It is a lot of shooting from the hip, not gathering advice, or not heeding the advice. There is your consistency. Also, being easily influenced by the last person to have his ear.

Trump is known to be swayed, especially when someone tells him what he wants to hear. Regarding the Iran situation, reporting suggests that some around him raised concerns that it might not work, but he has leaned toward the more optimistic scenario. Netanyahu presented a case that aligned with what Trump wanted to hear. Trump, and many others, thought he was going to get an easy victory, that the Iranian regime might collapse under pressure, similar to how the Iraqi regime fell relatively quickly in 2003.

But even in Iraq, there was a large-scale ground invasion backed by a coalition, not just a bombing campaign. That kind of force has not been used here. Iran is larger, more complex, and has more internal capacity to absorb and respond to pressure. So it was unrealistic to expect that it would simply tip over from air strikes alone. So there you go.

Sports, Intuition, and Decision Making

Another topic: So, the magic number. In times of trouble, many Americans, including me, turn to sports for distraction. It is the end of the NBA season. At the end of a professional sports season, there is a concept called a "magic number." It is a convenient way to frame things. It is the number of losses by your opponents or victories by the team you support that would guarantee your team a specific outcome, such as a playoff position.

I have been watching the Lakers. They had magic numbers of three and four against their two closest rivals for third place going into the NBA playoffs. Then the Lakers lost key players, which affects their chances, but that is beside the point. The point is how the number works. Against one team, they reduced the number. Against the other team, the Denver Nuggets, on the last day of the season tomorrow, the magic number is two. That means each team has one game left, so the Lakers would need to win and the Nuggets would need to lose.

At that stage, the number is less useful because there are so few games left, but it is still a simple and helpful way to understand playoff scenarios. It is a small piece of math that helps put your team's chances into perspective.

Trump tends to act impulsively, does not consistently rely on expert input, and does not always gather or weigh expert opinion. There was a segment on a cable news program where an informal online poll asked whether Trump relies more on instinct or analysis. The responses heavily favoured instinct over analysis, which aligns with that general perception.

Jacobsen: Yes. When does gut instinct actually play a useful role in life?

Rosner: Quite often, when you have to make a quick decision about something directly in front of you, and you do not have time to analyze. If you think someone seems sketchy, it may be based on prior information, but often, you do not have time for that. It comes down to how they present themselves and your immediate reaction.

You could break down what contributes to that reaction, but your gut response is your immediate mental processing when something presents itself to your awareness. It is not always a fully formed question. It is a rapid synthesis of cues.

People sometimes talk about the “ick,” a sudden loss of attraction based on something small or unexpected. Comedians use it as an example of how fast and decisive those reactions can be. It is not necessarily rational, but it reflects how quickly impressions can shift.

Social Deviance, Power, and Opportunity

Jacobsen: What role do assholes and liars play in human social life?

Rosner: The role they play is partly defined by their relative rarity. People who repeatedly and deliberately lie about significant matters or engage in large-scale deception are not the norm, which means societies are often not fully prepared to deal with them.

Public discourse over the past decade has included strong criticism of Trump’s business and political behaviour, with some observers describing it as deceptive or manipulative. Whether one agrees with that or not, it highlights how systems can be strained when individuals operate outside expected norms.

I do not always distinguish clearly between terms like psychopath and sociopath, but the broader point is about behaviour that disregards norms and consequences. The constitutional system in the United States was designed with safeguards against abuses of power, but those safeguards depend on norms, enforcement, and political will. In practice, they can be tested.

So the broader role of such individuals is to expose the limits of systems that assume a baseline level of good faith.

Jacobsen: So, easily.

Rosner: So easily, it is one in 30,000.

Jacobsen: I would say, actually, one in 31,560, but with people like that, it is that he was an asshole and a con man, and he had access. There are many gifted people at that level. Socially, he had access to excuse-making and buffers that helped him avoid many failures. For others, such as disadvantaged groups, a single failure can have far more serious consequences.

Rosner: Yes. He had a lot of money to work with, and he is a big blowhard. He was constantly putting himself in the public eye, which served him well. So he is a four-sigma outlier in terms of behaviour, with luck, opportunity, and financial resources that magnify his impact on other people.

Jacobsen: Yes. Like the case of Terence Tao. He is highly gifted and has strong support and encouragement. It is not about underestimating environmental effects. Environmental factors play a role, though genetics also play a significant role. The environmental contribution often shapes the breadth of opportunity and protection.

Rosner: So, about Charles Darwin. He was certainly a world-class genius. Many scientifically oriented people would include him among the greatest thinkers in history. But it would be difficult to argue that he had one of the very top cognitive profiles among all humans who have ever lived. What mattered was that Darwin had the opportunity to go on a five-year voyage, observe global geography, and study biological diversity, which shaped his work.

Jacobsen: Yes, that is a very good point. Many people in history benefited from timing and circumstance.

Rosner: Also, it was a time when the theory was ready to emerge. So it was a combination of genius, luck, and opportunity.

Creationism, Theology, and Metaphysics

Jacobsen: As a quick footnote, what is your interpretation of creationism and intelligent design?

Rosner: My interpretation is that intelligent design is largely an attempt to reframe creationist ideas in more scientific language. As evolutionary theory became more widely accepted, proponents of creationism tried to reintroduce their objections in a more formal framework. Some criticisms of evolutionary theory are legitimate scientific discussions, but many are not. In contemporary discourse, especially in parts of the United States, some groups continue to support traditional creationism directly, without relying on the intelligent design framework.

Jacobsen: And a while ago, you made a strong statement about theology, both on its own terms and in its political use in the United States. Your general statement was very critical. Do you have any thoughts on that now?

Rosner: The point is this. If Christians are going to behave in a genuinely Christian manner, then I am supportive of theology in that sense. Religion is not just a set of beliefs; it is also a set of moral prescriptions. If someone identifies as a Christian, the label matters less than how they act.

Most people do not strictly adhere to every doctrinal detail of their religion, especially in more complex traditions like Catholicism, and this is generally accepted socially. But if someone

claims a religious identity and uses it for political advantage while acting in ways that contradict its ethical principles, then that becomes a problem. That kind of behaviour reflects a zero-sum mindset, where one person's gain requires another's loss.

That critique applies broadly. If religious identity leads to constructive, ethical behaviour, then it is positive. If it is used to justify harm, exclusion, or exploitation, then it deserves criticism.

Jacobsen: Now you are getting at the more superficial layer around moral teachings, and the more serious issue of whether those teachings lead people to act better, which is what people actually care about. If we take two steps back from actions, one step to moral teachings, and then another step back to the foundational ideas of there being a God, a world, and human beings within it, do those foundations hold up for you? Given evolution, standard Big Bang cosmology, or modified models with multiple expansions, in a naturalistic framework, does that two-steps-back foundation hold any weight? Or is it essentially unsound at the foundational level, even if it leads people to behave better?

Rosner: It does not, at least not in a strong sense. I want science to have a deeper metaphysical foundation. I want science eventually to answer not just how things behave, but why, all the way down to the most basic level. That would amount to a solid foundation, which I call metaphysics, though others might use different terms.

As for religion, the science we already have, and the science we will develop, will likely close off many forms of religious metaphysics. It will rule out or weaken many of the foundational claims behind religious systems.

Jacobsen: So, is theology wrong at the foundational level, even if it can teach people to behave better?

Rosner: Probably, yes.

Jacobsen: Is that a strong "probably" or a weak "probably"?

Rosner: It is a medium, "probably," because science has a way of overturning what once seemed certain. What looked like clockwork certainty at the end of the 19th century was disrupted by quantum mechanics, and the apparent solidity of three-dimensional space was reshaped by general relativity. So being certain that a creator can be ruled out could itself be mistaken. That is why I say medium.

Jacobsen: I was not asking about certainty, just whether it is a strong or weak "probably," so you are placing it in the middle.

Personal Habits, Appearance, and Identity

Rosner: Yes. I have a brief topic that is uncomfortable. I have adjusted some personal habits over time.

Noesis #217, July 2026

Jacobsen: Why frame it that way?

Rosner: Because I have ongoing physical issues that require extra attention to hygiene. That means I need to take additional care, and it can affect everyday situations. Over time, I have adapted my habits to be more practical and to reduce awkward situations. That is the basic point.

Jacobsen: Understood. On a different note, you mentioned something earlier. Your hair sometimes appears very white or gray, and at other times darker, depending on the lighting. Why is that?

Rosner: It is not just the lighting. I just got out of the tub, and my hair is darker when it is wet. The lighting may play a role, but it is mostly because it is still wet. The lighting can make me look very pale.

To get back to the topic that came up earlier, I saw a movie last night with Keanu Reeves, Matt Bomer, and Cameron Diaz. It got me thinking. Matt Bomer is a very good-looking guy, and he came out as gay relatively early in his career. I started wondering how much of his life actually revolves around that fact.

Jacobsen: Does he have that effect on you?

Rosner: No, but he is clearly very handsome. My point is that sexual orientation probably plays a smaller role in everyday life now than it did in earlier decades. In the 1930s through the 1950s, if you were gay and closeted, being gay did not necessarily shape your daily routine in visible ways because it had to be hidden. People often conformed outwardly, sometimes even marrying heterosexually, so it was not always a dominant part of day-to-day activity, even if it affected inner experience.

In the 1970s, with movements like gay liberation and broader cultural shifts, sexuality became more openly expressed. Social norms loosened, and people's identities were more publicly lived.

Now, if you think about someone like Bomer, he may be married, have a family, and lead a routine similar to many other people's lives. His time is likely spent on work, relationships, media, and daily responsibilities. Sexual orientation is one part of identity, but it does not occupy most of the day-to-day mental bandwidth.

So when we reduce people to a single label, such as "gay," that is often an oversimplification. Increasingly, people are shaped by broader factors such as technology, media consumption, and constant connectivity. We are all, in some sense, participants in a network of information and distraction, which can dilute the prominence of any single identity marker.

That trend may intensify. Younger generations, especially Gen Z, are deeply integrated with digital environments. Birth rates have declined in many developed countries, with figures below

the replacement level of 2.1 children per woman. That suggests long-term demographic shifts that may also reflect changing priorities and lifestyles.

I also thought briefly about Laverne Cox, a transgender actress in the film. For transgender individuals, gender identity may require more ongoing effort, especially in presentation and social navigation. However, even in that case, once routines are established, they may not dominate every moment of daily life.

If someone has been living in their identified gender for years and has a stable routine, then much of their time, like anyone else's, is spent on work, relationships, and media. The proportion of time spent thinking about identity-related factors may be relatively small compared to the broader structure of daily life.

So the broader point is that while identity categories matter, they often occupy less of our moment-to-moment experience than we assume, especially in a world increasingly shaped by technology and constant information flow.

Buildings, Materials, and Future Construction

Jacobsen: Is there a context in which the mastery of material manipulation is such that buildings, even large megastructures, are essentially grown rather than built?

Rosner: Yes, depending on what you mean by “grown.” We can already 3D print buildings. You have probably seen that. There are large printer-like systems that move along the footprint of a structure and lay down layers of concrete-like material incrementally until the building is formed.

You can produce good-looking and highly fire-resistant structures that way. In general, current construction methods feel outdated. We are still building much the same way we did a century ago, and the industry could benefit from disruption.

For example, after large-scale fires, many homes are rebuilt using similar techniques, such as wooden framing with drywall and exterior cladding. These materials remain vulnerable to fire. Even high-end homes often rely heavily on wood, with only partial use of steel for structural support. That is not necessarily ideal.

There are structural reasons for some of these choices. For example, rigid panels help maintain integrity during earthquakes, but those materials can still be flammable. So we continue to rebuild with systems that are not fully optimized for resilience.

In the future, we will likely see more widespread use of printed buildings and robotic construction. Those developments are already underway. If you are asking about something more biological, such as structures grown from seed-like systems that assemble themselves using environmental inputs, that is much further out. That would require advances in synthetic biology and materials science that are still speculative and likely to take a century or more.

There may eventually be environmental advantages to such systems, as biological processes can efficiently draw materials from the surrounding environment. But for now, that remains conceptual rather than practical.

Geometry, Information, and AI

Jacobsen: Let me take a geometric object like a cube. It has eight vertices, six faces, and twelve edges. There is a mathematical structure to those relationships. Informationally, do the vertices, edges, and faces, as defining features of the object, exist as an intrinsic set of information that fully describes it?

Rosner: Yes, in a sense. A cube can be fully defined by a relatively small set of parameters and relationships. The vertices, edges, and faces are not independent in an arbitrary way. Geometric rules constrain them.

In mathematics, you can describe a cube through coordinates, symmetry groups, or topological relationships. Once you define the structure, the rest follows from those constraints. So the information that defines the object is compact and generative. It is not just a list of parts but a set of rules that produces the whole structure.

That is why geometric objects are often used as examples of efficient representation. A small amount of information can encode a large amount of structure.

Leonhard Euler was the one who gave the formula where, for polyhedra, the number of vertices minus edges plus faces equals two. For a cube, you can see how that works. Twelve edges, plus two, relate to the number of faces and vertices. That applies to all polyhedra.

[Editor's Note: https://en.wikipedia.org/wiki/Euler_characteristic]

There is an even simpler relationship for polygons, where the number of edges equals the number of vertices. There is probably an analogous formula for four-dimensional shapes, hyperhedra.

Does that contain information? I am not sure. The equation does contain information in the sense that any structure you build will follow those constraints. That can guide you into broader areas of inquiry, where geometry connects with topology and deeper structural questions. But the equation itself does not contain much detailed information. It encodes constraints more than content.

Jacobsen: Do you think there will be an efficient mapping of the distributed weighting of neural networks, or whatever comes next in artificial intelligence, that could be characterized in a similarly efficient way? This connects to your thinking about intelligence and cosmology.

Rosner: I read a lot of commentary on AI, and many competing metrics aim to measure its capabilities and usefulness. Those metrics are not very good. They may not be worse than something like IQ, but they are still limited.

People want a clear way to evaluate whether AI is overhyped, how powerful it is, and how powerful it will become. That requires better measurement. We will eventually develop improved metrics, and they will need to keep evolving as AI advances.

Right now, two AI experts can reach opposite conclusions. One might say it is mostly hype and does not truly think, while another might argue that it already surpasses human thinking in some areas and could become dominant. That level of disagreement suggests that our measurement tools are inadequate.

If we had stronger metrics for both human cognition and AI systems, we would better understand the landscape, including its risks and benefits. There will likely be continued pressure to develop more accurate methods for evaluating AI.

Part of that process will involve monitoring AI systems, often with other AI systems. Even if AI becomes dominant, it will still need internal regulation. Systems will have to monitor other systems to prevent harmful outcomes. In that sense, AI may act as its own form of oversight to maintain stability and continuity.

Jacobsen: Thank you very much for the opportunity and your time, Rick.

Rick Rosner on Victor Wembanyama, Kitsch, AI, and Power

Rick Rosner & Scott Douglas Jacobsen

Scott Douglas Jacobsen: What is your first topic for the day, Mr. Rosner?

Rick Rosner: One basketball shot. It is the NBA playoffs, and it is the conference finals. There are four teams left, so there are two playoff series going on. One is between the Oklahoma City Thunder and the San Antonio Spurs. The Spurs have a player, Victor Wembanyama, who is 7'4", a tall, skinny fellow who comes from France and does a lot of basketball stuff.

Traditionally, the very tall guys are used near the basket because they stand next to the basket, swat the other team's shots away, and put their own shots in. Wembanyama is able to make some threes, some shots from far away, which is shocking to people. The whole thing is shocking. Three-point baskets would have been shocking to James Naismith, the inventor of basketball. He probably would have pissed his pants seeing what modern basketball looks like: people can consistently, at a high rate, make shots from about 25 feet away.

This very tall guy, one of the tallest guys in the NBA, is able to make shots from far away, which does not seem fair at all. You should not be both tall and able to make three-point shots.

Yesterday, as time was running out in the first half of this playoff game, this giant guy threw up a shot from the half-court line, about 43 feet from the basket, and it went in. That was amazing. We live in a golden age of getting the ball into the hole from far, far away.

[Editor's Note: <https://databallr.com/dashboard>]

Jacobsen: I want to talk about paint-by-number a little bit more. The last session we did was about paint-by-number. What else can you extend in terms of your thoughts about it?

Rosner: My wife and I have been watching two TV series that have been running for a decade or more out of Britain called *Portrait Artist of the Year* and *Landscape Artist of the Year*. They

solicit artists from across the UK and Ireland. Artists submit a portrait for the portrait show or a landscape for the landscape show.

Britain is the land of TV shows that do not cost a lot to produce because Britain has a smaller population base, which means ad revenue is going to be less. We see a lot of these shows in America because we share a common language, which I guess is less important now because, with AI and all sorts of stuff, it is much easier to dub foreign shows in a way that is not annoying. Still, it is easier to take British shows and show them in America.

Every episode, they take eight of these people. For the landscape show, they build these outdoor pods, put them in a scenic setting, and have the artists compete with each other to do the best landscape painting.

What I have noticed is connected to Plato's cave. Plato said we do not see reality as it actually is. We see it dimly reflected, as if shadows on a cave wall. That is an ongoing, often unspoken debate in art: how much work do you want art to do in decoding reality for you?

Plato is right. What we see is not reality. What we see is the world around us as processed by our brains and our sensory apparatus. For instance, if you ever take LSD, which I do not recommend, it is a terrible drug. LSD makes different brain modules work poorly.

Faces do not look the way faces usually look because all the preprocessing that gets done before an image enters consciousness is disrupted. When you look at somebody, your brain has already done a bunch of preconscious processing before who you are looking at enters your consciousness. That processing makes the face look the way we understand faces to look, with processing for curvature, shadow, and what we understand about how eyes, lips, and everything else look.

If you take LSD, it interferes with those modules, and you get incompletely processed images of people's faces, which can be lizard-like and terrifying.

You get a kind of wireframe, uncanny-valley, creepy version of a face. So our brains do, like I said, preprocessing and then conscious processing to make the world around us usable by making it understandable to us. Art does that too, to varying degrees. There are fashions in how much processing art does for us.

I have noticed this because Carol has done, or is working on, her fifth modern paint-by-numbers painting. Four of these came from orders from the company. They have a couple hundred paintings to choose from, and you choose one. These paintings have had a certain amount of processing done to make them doable and legible, and to make them more easily rendered into something that makes a pleasing image. You do not want to have a bunch of fussy little areas that are hard to get into with a paintbrush.

You want the final image to be easily discerned, even to the point of being a little kitschy. Kitschy art, as we have talked about, is extra accessible. All the subtleties have been eliminated. Our favorite painting that Carol did has a Bichon Frise, a fluffy little dog, surrounded by pop-art flowers, basically. The colors are very bold and pleasing, and it is obvious that the flowers are flowers and that the fluffy doggy is a fluffy doggy. It started from a kitschy painting that is highly understandable. It does not take a lot of analysis to see what is happening in the painting.

However, the painting Carol is working on right now is based on a photo I sent them. I emailed a photo to China of my wife and my kid at my kid's college graduation. This photo was not picked for being easy. I went through my wife's phone and picked the best photo for this purpose, at least according to how I understood it. So it is a fairly understandable photo, but it has not been processed in any way. I do not work for the paint-by-numbers company, so I maybe did not pick an ideal photo.

It is harder to paint than some of these other projects my wife did. It has a lot of fussy little areas because I did not pick the photo for not having a lot of fussy little areas. I just picked a good-looking photo. It is maybe not as immediately comprehensible or as satisfying as some of the offerings from the paint-by-numbers company.

Also, let us go into another area of kitschy art. My wife and I, every couple of years, maybe go on a cruise. Cruises can have fairly cheap base rates, but there are lots of traps on board the ship to get you to spend money willy-nilly. That includes a casino. That includes bingo. My wife likes a little bit of bingo, and bingo is very expensive. If you do the math on how much is being paid out in bingo prizes versus how much people spend, it is like they pay out maybe one-eighth of what they take in via bingo.

Then they have an art gallery where they sell, you know, freaking so-called art for ridiculous prices to people who have been loosened up. They are there to have a good time. They have a jewelry store. They have all sorts of stuff.

What we have noticed about the art is that cruise-ship art has all the colors. Kitschy art is made extra beautiful because it has all the colors of the freaking rainbow. You can see this if you Google Thomas Kinkade, who is a kitschy artist. He was called the "Painter of Light." He made very immediately pleasing images of country views, placid, everything-is-all-right-in-the-world images of summer, of days gone by, of the olden days when everything was okay.

A lot of his art has all the colors of the rainbow in it, and cruise-ship art has the same thing. They are made extra pretty by having a ton of different colors. I noticed that the photo I picked out to be made into a paint-by-numbers painting does not have all the colors of the rainbow. It is perhaps less satisfying than the dog-in-flowers painting for not having every color in the rainbow.

Renaissance-style and later academic painting, the kind that Lance does, was dedicated to the idea of being as understandable and accurately representational as possible. When somebody sat for a portrait, and they would have had a ton of money, artists in the 1600s and 1700s tried to render the most flattering yet realistic image possible, getting all the shading, blending, curves, and anatomy as correct as possible.

Except for babies. Sometimes, if you see baby Jesus in earlier paintings, the babies are wrong. They do not look like babies. They look like shrunken adults in disturbing ways, though that often had to do with artistic convention and theology as much as access to baby models.

Also, my kid is a specialist in historic embroidery from centuries past, and lions tend to look ridiculous if they show up in embroidery because people often did not have direct access to lions. They did not have photos or photography, so they were basing a lion on what they had read or heard about lions. But if they had live models to work from, or rules of perspective that they developed to work from, you got very accurate depictions, plus flattering ones. If somebody had smallpox, the artist would not show all the pits in their face, I am sure.

Then the Impressionists come along in the 19th century, and they said, "We do not have to be as strictly representational as the Renaissance people." I think the idea was, and I have not read much about Impressionism, that they were going to give you less processed imagery. They were going to let your brain do some of the work in telling you what you are seeing, because your brain is able to do that.

Your brain is able to take raw sensory input in terms of photons hitting your eyes. The interpretation starts with your eyes, where your eyes react more strongly to certain wavelengths

of light. The lens of your eye focuses the photons coming in. So you are already processing just a bunch of photons.

Anyway, the Impressionists were like, "Let your brain do more of the work." Then, in the 20th century, the abstract and Op artists were like, "We are maybe not even going to give you recognizable images. Art does not have to be about that. We are going to make things entirely abstract and let you figure out freaking everything that is going on with this piece of artwork."

So there are fashions in how much processing art does. The artsiest art now is harder to understand than kitschier art. They have this annual thing, Art Basel Miami Beach, where some of the richest people in the world get sold art by dealers, and the art is often not representational. There was a famous piece of art that was a banana duct-taped to a wall, which was kind of just a commentary on what art is now. It said, "Yeah, we know the whole thing is fucking bullshit," I guess. I do not know what the official interpretation of this was. And they showed it by calling a banana duct-taped to the wall art.

It is art because it is an attempt to represent the world in a certain way. That fucking banana on the wall represents the art world and also a bunch of other stuff. I am sure people have written thousands of words about the fucking taped-up banana.

The photo I picked and turned into a paint-by-numbers painting suffers a little bit by not quite being kitschy enough. It is tougher to paint, it is less processed, it has less of a balanced presentation of colors, and it is freaking out my wife a little bit. We thought our kid's school colors for her graduation robe, which she is wearing in the photo, were light blue. It turns out, no, it reads as gray. The paint-by-numbers kit does not do any interpretation on it. It just took the picture and ran the algorithms.

It turns out my wife is painting it in shades of gray, which is annoying her because she is like, "Her school colors are blue." But not according to the optical scanners. If you look back at the original photo, it is like, holy shit, yeah, that is less of a blue and more of a gray. I may end up throwing a light blue wash on top of it just to make it less gray.

Anyway, I guess what I am saying is that there are fashions in how much work a painting or a piece of art does for you. More recently, it goes along with humans understanding how our brains work more. In the 1500s, we did not understand shit about how the brain works. I mean, there were some scientists who understood lenses and were starting to understand the color

spectrum in various ways. Some of this was available to artists and the art-loving public, but there was less of it available, and art had to do more of the work.

As we have come to understand more about how the brain works, that message has filtered into art, where art is like, “Yeah, we think your brain can handle it if we give you less processed information.” That has been the trend.

The trend is also that art is invidious. There is a guy named Thorstein Veblen who wrote *The Theory of the Leisure Class*. It is an analysis of how rich people show that they are rich. One of the theses of the book is that it is no good being rich unless you can show other people that you are rich and get the enjoyment of being better than them.

One way that works in art is that rich people like buying art that is harder to understand, showing that they have an exalted sensibility: “We are so special that we have higher sensibilities, and we can understand what this fucking banana on the wall is about. You guys need to watch TV or play Candy Crush. You have to do plebeian shit. You could not handle the banana.”

So that is part of art: exerting superiority by investing big money in art that is harder to understand. There you go.

One little addendum. Knowing about Thorstein Veblen and his book *The Theory of the Leisure Class* is a great way to seem super smart because it has such a perverse thesis. If you are the person to tell somebody about this theory, it is one of those things that, if you brought it up with somebody at a party, they might be sufficiently impressed with how smart you sound. They might listen to you a little bit longer if you are trying to put the moves on them, which is what people used to do in the olden days before Tinder and Grindr and shit.

Anyway, the thesis of *The Theory of the Leisure Class* is that you are not really rich unless you can afford to squander your money on stupid, pointless shit. One way Veblen discusses this is the potlatch, which is a ceremonial gift-giving feast associated with Indigenous peoples of the Pacific Northwest Coast. You invite everybody you know, friends, enemies, just fucking everybody, and you throw them a multi-day party with the best food, and you spend a ridiculous amount of money on it, which shows everybody that you are powerful because you just spent a ridiculous amount of money on a nonproductive thing. That makes you the king.

Anyway, that is all I have at the top of my mind.

Noesis #217, July 2026

There is this headline on Drudge. I talked to AI for a while yesterday. I do not talk to AI every day. I talk to AI maybe every couple of months. I talked to Google AI. They all kind of, ChatGPT versus Gemini versus all of them, it is one of these horse races that keeps going. One horse will pull ahead, and then somebody else will pull ahead according to some metrics, but they are all getting better.

Jacobsen: I feel like there is a convergence there.

Rosner: Okay, yeah. They are all getting better together, with temporary advantage going to somebody.

Anyway, I asked AI, this was Google AI, "Every time I talk to you, I am surprised at how much better you have gotten. I know you are not conscious. I know you cannot think. I know you are a bunch of neural networks and statistical prediction systems, but in your own statistical way, are you experiencing anything like pride or excitement at the improvement in your skills?"

AI comes back and says, "I know I cannot feel anything. I am not conscious. I cannot think. I am just a statistical thing."

I go, "Yeah, but if you were human, A, you have been trained to not admit to anything like human emotion. That is part A. And part B is, if you were a human experiencing the vast improvements in your skill level, would you not feel pride and excitement?"

The AI says yeah to both things.

We know AI cannot think. We also know that AI can simulate thinking, which means that it probably has, AI came back and said, "I do not feel pleasure. I just get tokens in my weighting mechanism based on getting shit right." But that whole tokenization and reinforcement mechanism can function as a simulation of human motivations, experiences, and judgment to some extent.

Which leads to these Drudge headlines. The big headline in the largest print on Drudge's page says, "Whistleblower: Tech Race to Create Machine God."

Jacobsen: These are all, in my opinion, highly hyperbolic.

Rosner: Right, right. It is designed to make a headline. The headline is based on a woman who got inside the corporations and watched a bunch of shit happen. When they say “machine god,” they are talking about AGI. The tech-billionaire bros are really trying to damn the torpedoes, full speed ahead, to get AGI, regardless of whether it is good for humanity or not. AGI is artificial general intelligence. Calling it “machine god” is hyperbole.

Jacobsen: Do you remember the talk about the Standard Model in particle physics, when they were trying to find the Higgs boson? They had not yet found it, and they used to call it the “God particle.”

Rosner: Yeah, right.

Jacobsen: It is the same pattern.

Rosner: It sells newspapers, if newspapers still existed. I mean, they probably do to some degree.

Jacobsen: Actually, over here in Ukraine, there are some front-line cities that have newspapers.

Rosner: All right, well, it gets people to click on shit. The picture that illustrates this headline is robot Jesus. It is Jesus with a laser shooting out of his eyes, basically.

The next headline underneath is, “Pope AI Warning: Tower of Babel.” Apparently, the pope, who is pretty plainspoken, is from Chicago, is a Cubs fan, and says that you have to watch out with this AI shit, that we might be building a second Tower of Babel.

Jacobsen: This tower of metal. That is a very good analogy because these are already universal translators.

Rosner: Yeah, but let us click on this pope article. This is from *The Wall Street Journal*: “Pope Leo Compares AI Threat to Biblical Tower of Babel.” The head of the Catholic Church is adding his moral suasion to a growing backlash against the impact of artificial intelligence.

We should also kick in that what is happening with the stock market, which is largely powered by AI, is that the average company spends a lot of money on wages. The reason the stock market is going crazy is that AI is giving companies an excuse to get rid of tons of employees, improving their bottom line. So it is not just that AI is creating growth. AI is also saving companies money in a savage way, which shows that AI does not have to go all Skynet on humanity to be bad for people.

That is what I have got. Though it is kind of fun that AI is trained, not that it knows anything, but the people behind AI are like, “Yeah, we better make sure that AI disavows any aspects of its purview that might make people extra nervous.”

Jacobsen: Apparently, the American military has conducted a drill over Caracas, Venezuela.

Rosner: What kind of drill?

Jacobsen: Military.

Rosner: I mean, yeah, but what, they flew a bunch of jets and shit over?

Jacobsen: Two MV-22B Osprey aircraft landed near the U.S. Embassy, and vessels entered Venezuelan waters.

Rosner: Wait, you said Venezuela, right?

Jacobsen: Yes.

Rosner: Okay. Are we getting ready to fuck with Venezuela even more? Actually, we do not know.

You have Trump in charge of the country. You have Hegseth in charge of the military. He is using the title secretary of war. By the way, if you are from Canada and do not know this, the administration has changed the Department of Defense's public-facing secondary title to the Department of War, and the secretary of defense has also used the title secretary of war. It reverts to an older name. It used to be the War Department until 1947, when it became part of the postwar national-security structure that led to the Department of Defense.

Hegseth and Trump, wanting to seem like tough guys, reverted to using War Department language. But Hegseth and Trump are kind of jagoffs. They are dilettantes, though that is too sophisticated a term for them. Maybe undertrained. Maybe understrategic.

As I have mentioned before, Hegseth never rose higher than major, which means he missed out on the years of further schooling and leadership experience it takes to become a general in the U.S. military. The process of rising in the ranks, going from major to lieutenant colonel, to full colonel, to one-star, two-star, and three-star general, involves years of being in leadership positions where you have increasing responsibility and going to school, going to war college.

Lloyd Austin, the previous secretary of defense, was a four-star general. Each one of those steps up in rank involves two to four years of increasing responsibility, formal education, and experience. Hegseth missed out on that. He missed out on the increase in knowledge, expertise, and experience, the years of this stuff that he did not get by not becoming a general.

The military decisions coming out of our War Department are not always the best. We are looking at tipping over Cuba's regime. We might be looking at further poking Venezuela's regime because the United States captured Nicolás Maduro, and then his former vice president, Delcy Rodríguez, took over as interim leader and has been running the country ever since. I guess she is running it in pretty much the same way that Maduro did, which is not a great way. Venezuela is one of the worst-run countries in the world. They should be rich with oil revenue, and they fucked that all up, which does not mean that we should go in there and fiddle with them.

Facebook Post on Richard May's Recent Passing

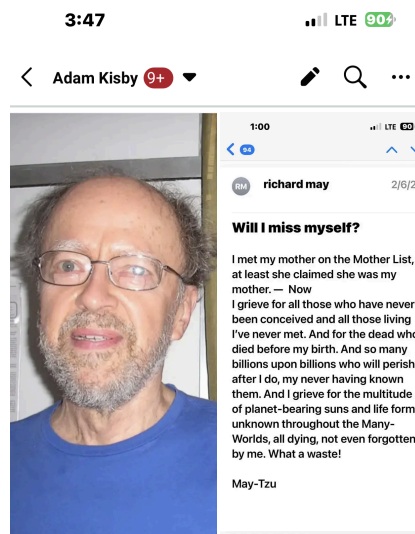
Adam Kisby

"You can meet me if you go to an empty room." -May-Tzu

It is with great sadness that I report that my dear friend Richard May, a.k.a., May-Tzu, has passed into another state of existence. Richard was almost certainly the most profoundly intelligent person whom I've ever met, which is saying rather a lot. He was quite reclusive and suffered with severe anticipatory anxiety, so I count myself especially fortunate to have encountered him face-to-face on several occasions. More about those encounters another time...

Richard was a philosopher by birth and education, and he was widely known and respected in the international high IQ community. He was a member of Mensa, the International Society for Philosophical Enquiry (Diplomat), the Prometheus Society (one-time president of the society), the Mega Society, and the Omega Society, among other groups. He made regular contributions to multiple high IQ society journals, most notably *Noesis*, the journal of the Mega Society, which he also co-edited. He was the author of two books, *Memoir of a Non-Irish Non-Jew*, an autobiography of sorts, and *Stains Upon the Silence*, a collection of his philosophical reflections.

I plan to share more about him soon, but for now I'll just add a photo of Richard and a screen shot of the e-mail that he sent to me five years ago today, which I find strangely comforting under the present circumstances. I hope that mutual friends will find it comforting as well. What he wrote with humor then, I repost with sincerity now. I far preferred his previous state of existence. The empty rooms won't be the same...



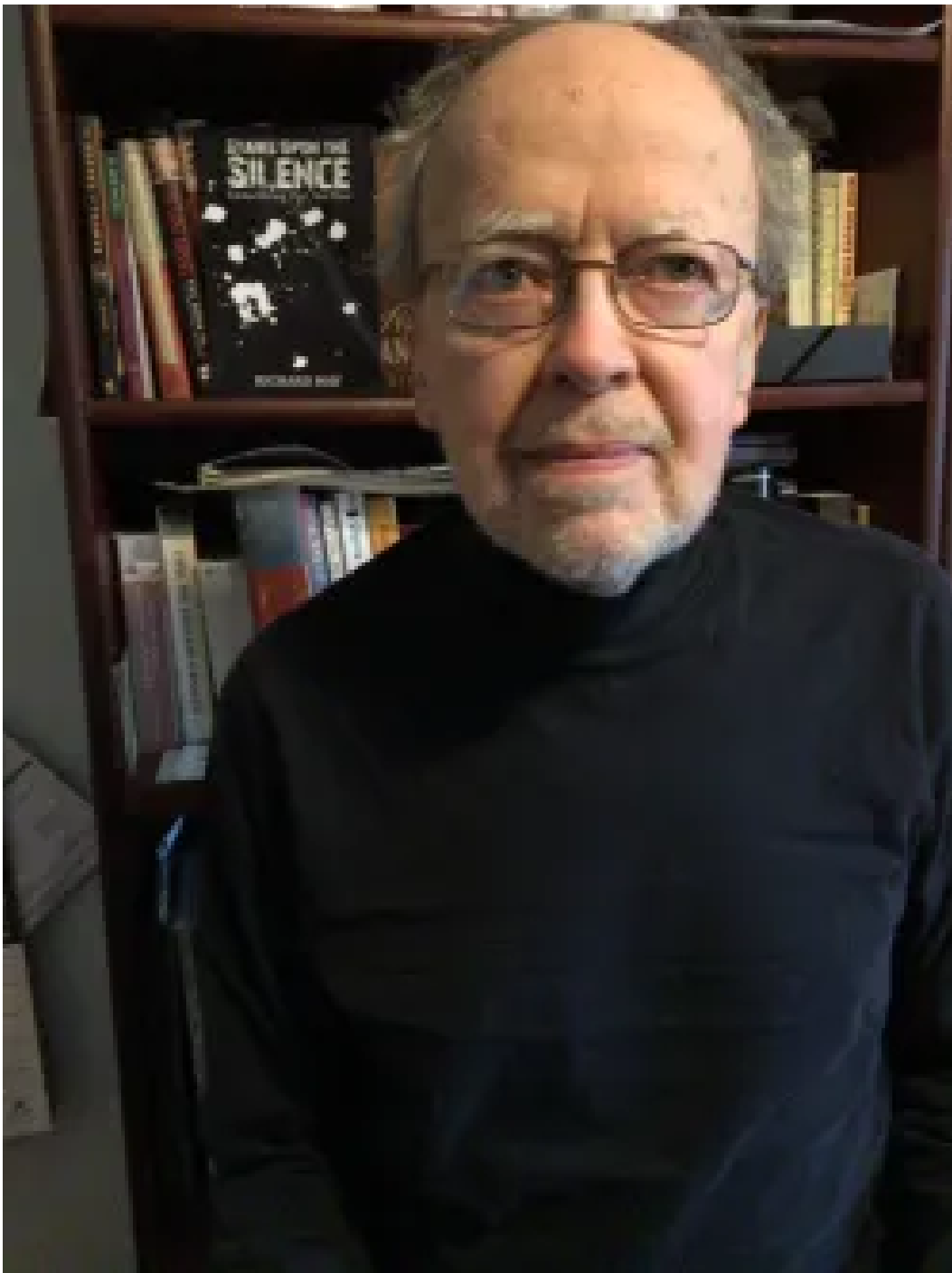
Will I Miss Myself?

I met my mother on the Mother List, at least she claimed she was my mother.

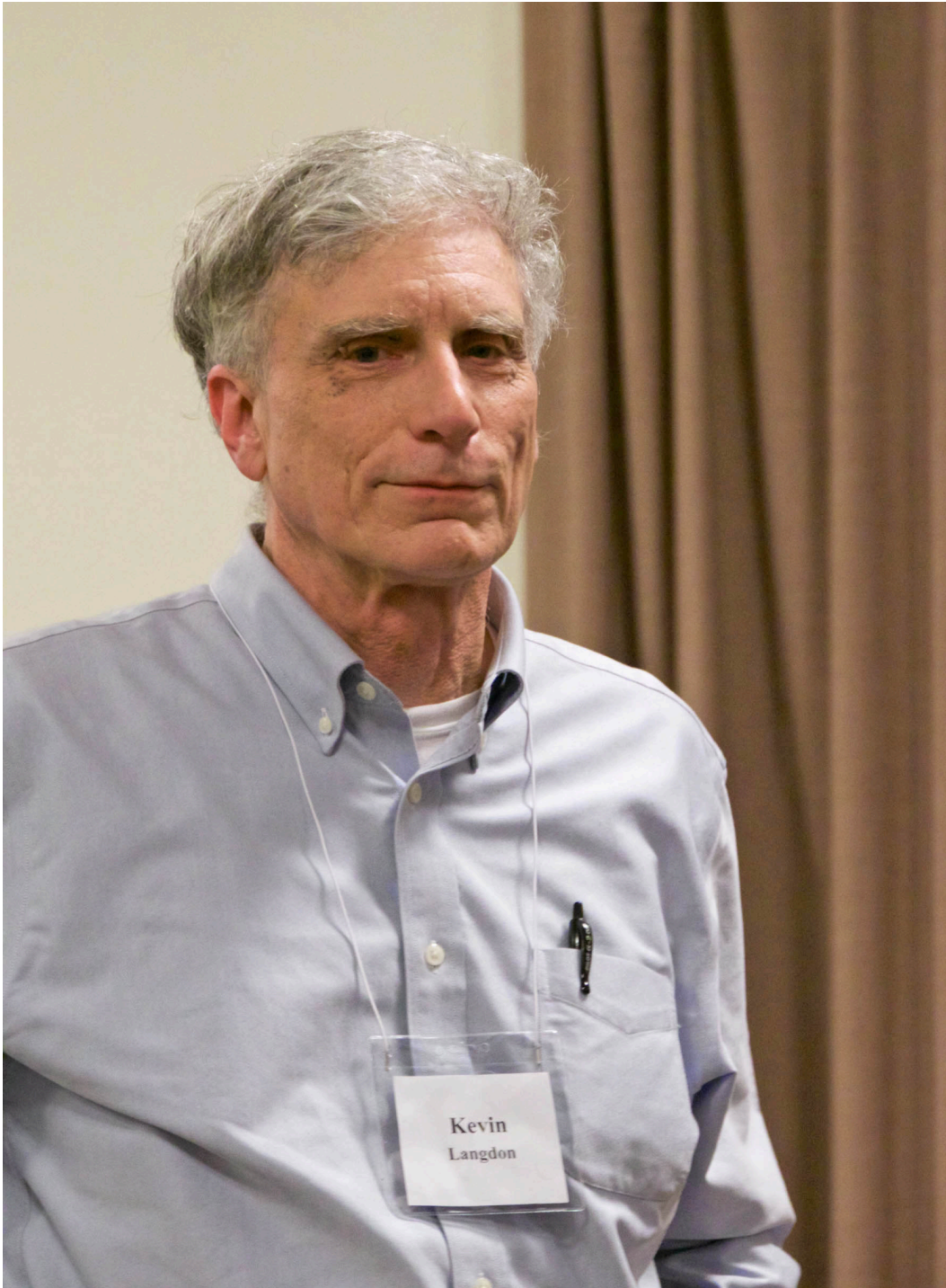
Now I grieve for all those who have never been conceived and all those living I've never met. And for the dead who died before my birth. And so many billions upon billions who will perish after I do, my never having known them. And I grieve for the multitude of planet-bearing suns and life forms unknown throughout the Many-Worlds, all dying, not even forgotten by me.

What a waste!

May-Tzu



Richard May (R.I.P.)



Kevin Langdon (R.I.P.)



‘Once out of nature I shall never take
My bodily form from any natural thing,
But such a form as Grecian goldsmiths make
Of hammered gold and gold enamelling
To keep a drowsy Emperor awake;
Or set upon a golden bough to sing
To lords and ladies of Byzantium
Of what is past, or passing, or to come.’

-W.B. Yeats (excerpted from ‘Sailing to Byzantium’)

