ISSUE 51 JULY 1990

NOESIS

THE MONTHLY JOURNAL OF THE ONE-IN-A-MILLION SOCIETY

Contents

Publisher & Editor

Ronald K. Hoeflin
P. O. Box 7430
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EDITORIAL

Ronald K. Hoeflin
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Numbering of issues: I received Ray Wise's "Issue 53" just prior to putting out this issue. Since that would leave a 3-issue gap, I ask all members to renumber his issue "Issue 49" and redate it "June 1990." The last issue put out by C. M. Langan was "Issue 49," dated "May 1990."

New procedures for publishing "Noesis": Starting with this issue, I will be responsible for reducing, reproducing, stapling, and mailing all issues of this journal. But the material in each issue will be supplied by several assistant editors. I have recruited three members so far for this purpose. You will be added to this list if you wish. Each assistant will be responsible for supplying me with ten pages of typed material every few months. Thus, George Dicks will be responsible for the August issue, Richard Sterman for the September issue, and C. M. Langan for the October issue. I will interject an issue of my own from time to time in order to take care of the society's business. Other members who do not wish to be responsible for an entire issue may submit comments at any time to any of these editors. Their addresses are as follows:

George Dicks	Richard Sterman	C. M. Langan
198 Sturm Street	3829 Encino Hills Place	P. O. Box 131
New Haven, IN 46774	Encino, CA 91436	Speonk, NY 11972

Typed material should have margins of 1.25 inches on both left and right sides, leaving 6 inches of text, which I will reduce to 4.5 inches. I will also be responsible for the outer cover and the page numberings, as you see them in this issue. After the October issue, if there are no additional volunteers to serve as editor, I will do the November issue, Dicks the December issue, Sterman the January issue, Langan the February issue, etc.

Dues: Since I will be responsible for reducing, duplicating, and mailing out each monthly issue, each member who wishes to continue as a member for the July through December period should send me a $10 check now, regardless of any funds that you sent or issues that you published at your own expense prior to now. Make payable to "Ronald K. Hoeflin." I plan to make this journal available to non-members for this same subscription fee of $10 per 6-month period. Deadline for receipt of dues is September 30, 1990. Those who do not submit a $10 payment by then will not receive further issues of the journal until they can make this payment.

Annual meeting: If anyone is willing and able to attend an annual meeting of this society within the next month or two in the Los Angeles area, please contact me immediately at (212) 582-2326. You might also contact Chris Cole, who would host such a meeting if enough interest is shown, at (714) 720-1761 (evenings) or (714) 727-3341 (daytimes).

Editor's note: Richard Sterman achieved a perfect score on my Titan Test and missed just one item on my Mega Test, giving him a 190 or better IQ on both tests (the 99.999999 percentile or one-in-100,000,000 level or rarer). The following is his (apparently unsuccessful) attempt to solve a problem I had posed in my Trial Test "A", preliminary to the Titan Test, involving finding the maximum possible number of pieces (completely bounded volumes not further subdivided) created by the interpenetration of a tetrahedron and four spheres. That problem proved to be too difficult for inclusion in the Titan Test and remains unsolved to this day. The hardest problem in the Titan Test, involving the interpenetration of a right circular cylinerr and two right circular cones, was solved to my satisfaction by only one of the 75 people who attempted Trial Test "A", Dean Inada, and so Dean's comments on Richard Sterman's attempt to solve this problem carry strong weight, in my view see the last page for his refutation of Sterman's solution. I will never use the tetrahedron-and-spheres problem in any intelligence test in view of its difficulty, so I have not censored any information concerning that problem, but since the cylinder-and-cones problem does appear in my Titan Test, I have deleted a few of the numbers Sterman gives regarding that problem, in case any of you have not yet attempted that test but might do so at some future time.

3829 Encino Hills Place
Encino, CA 91436
(213)471-5466
June 4, 1990

Dear Mr. Hoeflin,

Thank you for your notification of my Titan Test score, which my family and I found very exciting.

While question 41 was perhaps the most difficult mathematical problem on the test, I was haunted by question 35 (two cones and a cylinder), sketching many different configurations even after I sent you my answer sheet. I think I developed some techniques while working on that problem that are effective in solving the problem involving four spheres and a tetrahedron.

As with two-dimensional interpenetration problems, the way to maximize the number of regions in 3-D interpenetration problems is to maximize the number of points of intersection between the boundaries of the interpenetrating shapes. In 2-D problems, the number of regions equals the number of intersection points plus one (but only if all regions are genus zero). In 3-D problems the number of regions equals the number of intersection points plus two times the number of interpenetrating figures, minus one. There is a maximum of -- points of intersection among the surfaces of the cones and cylinder in question 35. (Though I was not entirely sure of this until your letter arrived.)

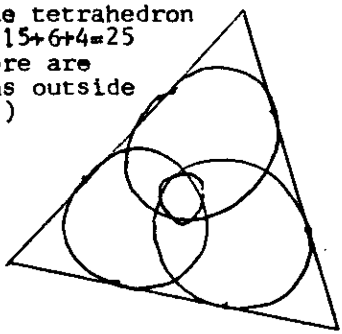
The techniques I tried with good, but not 100% conclusive, results on problem 35 have led me to a solution of the sphere-tetrahedron problem of which I am fairly confident (and hoping there aren't pitfalls I've overlooked). Here is my chain of reasoning:

Four spheres can have a maximum of 8 intersection points and a max of 15 regions. From any angle, a plane can slice through a maximally-intersecting set of 4 spheres, forming an additonal 12 intersection points. A tetrahedron can easily be situated so that each of its faces forms 12 intersection points with a set of 4 spheres (that is, a flat surface and the surfaces of two spheres meeting at each intersection point). So, it's fairly obvious that 4 spheres and a tetrahedron can be situated so that there are 8 intersection points where the surfaces of 3 spheres meet, and 48 intersection points where the surfaces of two spheres meet a face of the tetrahedron.

What's trickier is determining how many intersection points can be formed by the edge of a tetrahedron touching the surface of a sphere. Exactly one sphere can be tangent to all six edges of a tetrahedron, forming nine regions. An unlimited number of other spheres can each be tangent to four edges of the tetrahedron.

(Spheres can be tangent to five edges of a tetrahedron whose edges are of unequal length, but such arrangements offer no advantantages in maximizing the number of intersection points. A pleasingly symmetric solution to the problem has a sphere situated close to each vertex and tangent to only those three edges meeting at its particular vertex. Such an arrangement yields a maximum of 77 regions, with each face of the tetrahedron looking like this:

The inside of the tetrahedron is divided into 15+6+4=25 regions, and there are 4 x 13=52 regions outside the tetrahedron.)



June 5

A less-symmetric solution that yields more regions starts with a sphere tangent to all six edges of a tetrahedron and adds three more spheres each tangent to four of the tetrahedron's edges, yielding 83 regions. I hope and think that this is the maximum number of regions that can be obtained from the interpenetration of four spheres and a tetrahedron. A diagram shows the intersection of four such spheres with the faces of a tetrahedron. I redid the diagram with hexagons instead of circles to show some of the intersections more clearly. Inside the tetrahedron there are 4 + 18 + 17 = 39 regions, and outside there are 44 regions, for a total of 83.

June 6

After playing with this for a few days, I can see that 2-D diagrams aren't entirely adequate for determining the number of 3-D regions, making this an extremely difficult problem to visualize. However, I'm now quite sure that 83 regions is a maximum for this problem, as I hope the additional colored diagrams will help show.

In this solution, there are 8 + 4x12 + 6 + 3x4 = 74 points of intersection among the spheres and tetrahedron. It seems to be an inescapable rule that as long as shapes and intersections are topologically simple, $P + 2S - 1 = R$, where P is the number of intersection points, S is the number of shapes, and R is the number of regions (for 3-D problems).

So, visualization aside, the $P + 2S - 1$ rule makes this a fairly well-behaved problem, though, like many of the problems on the Titan Test, it chews up a lot of time. Please let me know how other people approached this, and if my work is convincing.

PERSONAL INFO:

Though I feel there is much to question about the concept of IQ, my self-esteem has gone up and down with my scores on aptitude tests. In elementary school, I got scores of around 150 on IQ tests. Discovering my scores and feeling that they weren't adequate for me to be a world-class thinker, I didn't take my classes or myself very seriously. Later, I found out more about statistics and IQ, and learned that group-administered IQ tests don't measure much above 150. After scoring well on ETS tests, I worked tutoring other people on how to take the SAT, etc., and I began to try to maximize my mental potential. About a year ago, I started to collect library cards and now have over 40 cards good at over 500 southern California libraries, giving me access to tens of millions of books, which was helpful in solving many of the verbal problems on the test. I try to read a book a day, which means that often I have to resort to reading trash or trivia, but I've managed to read about 1400 books in the last 4 years.

I'm very interested in cosmology, especially Stephen Hawking's imaginary time and baby universes, but I dislike high-energy particle physics, finding it makes false claims of elegance.

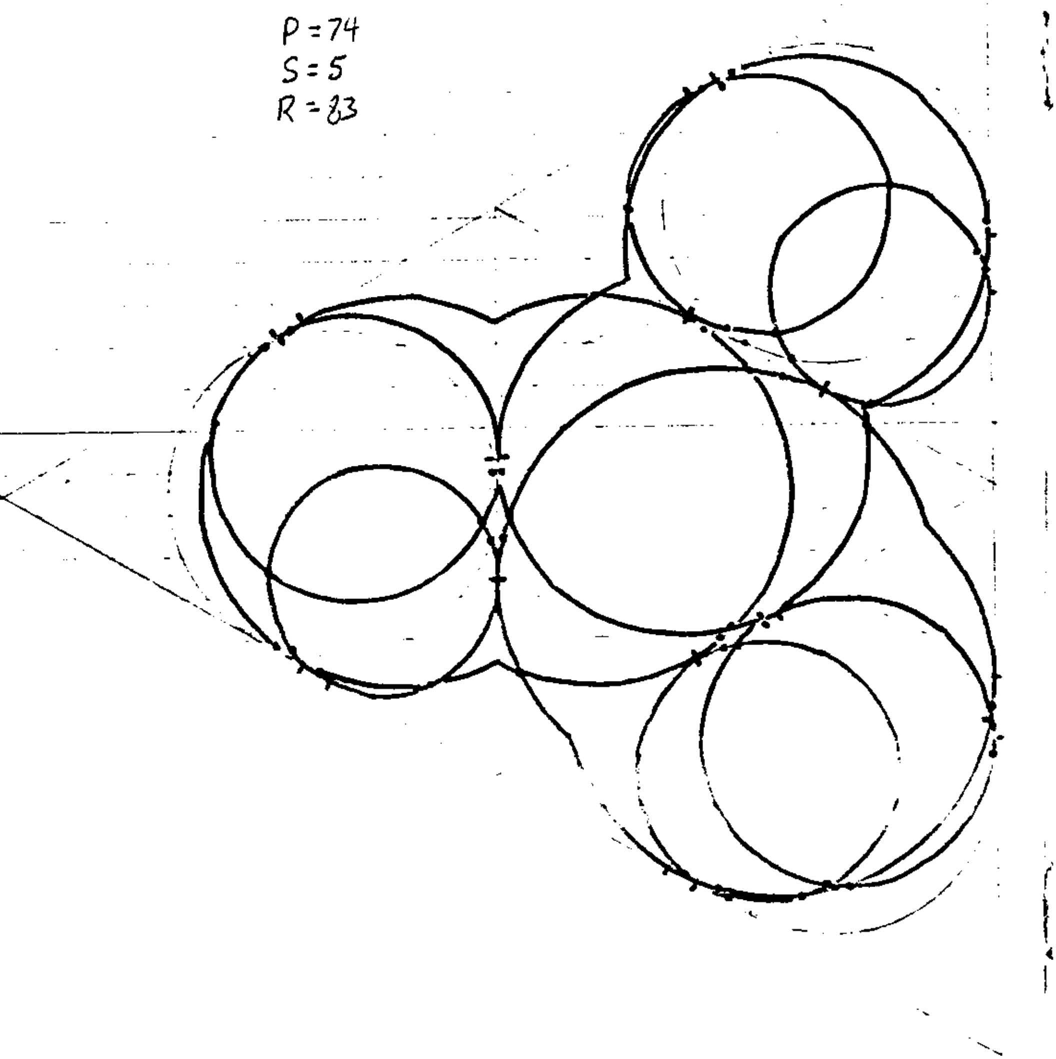
Thanks again for your letter.

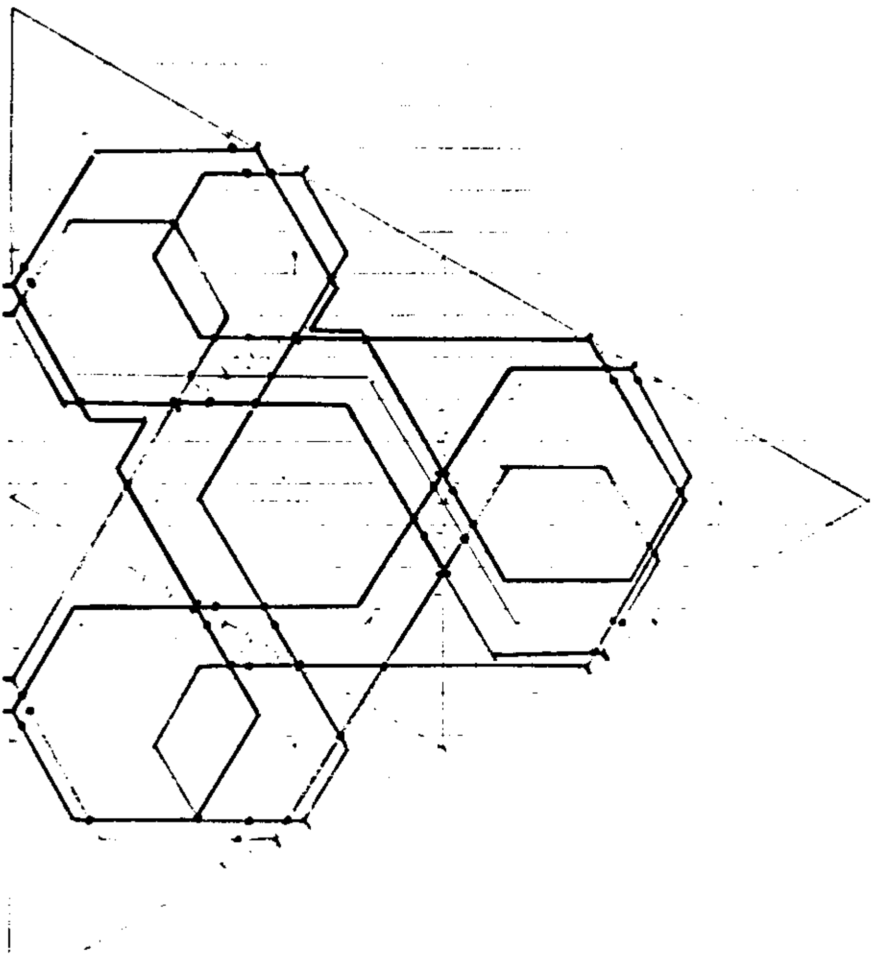
Best Wishes,

Richard Sterman

Richard Sterman

$P = 74$
$S = 5$
$R = 83$

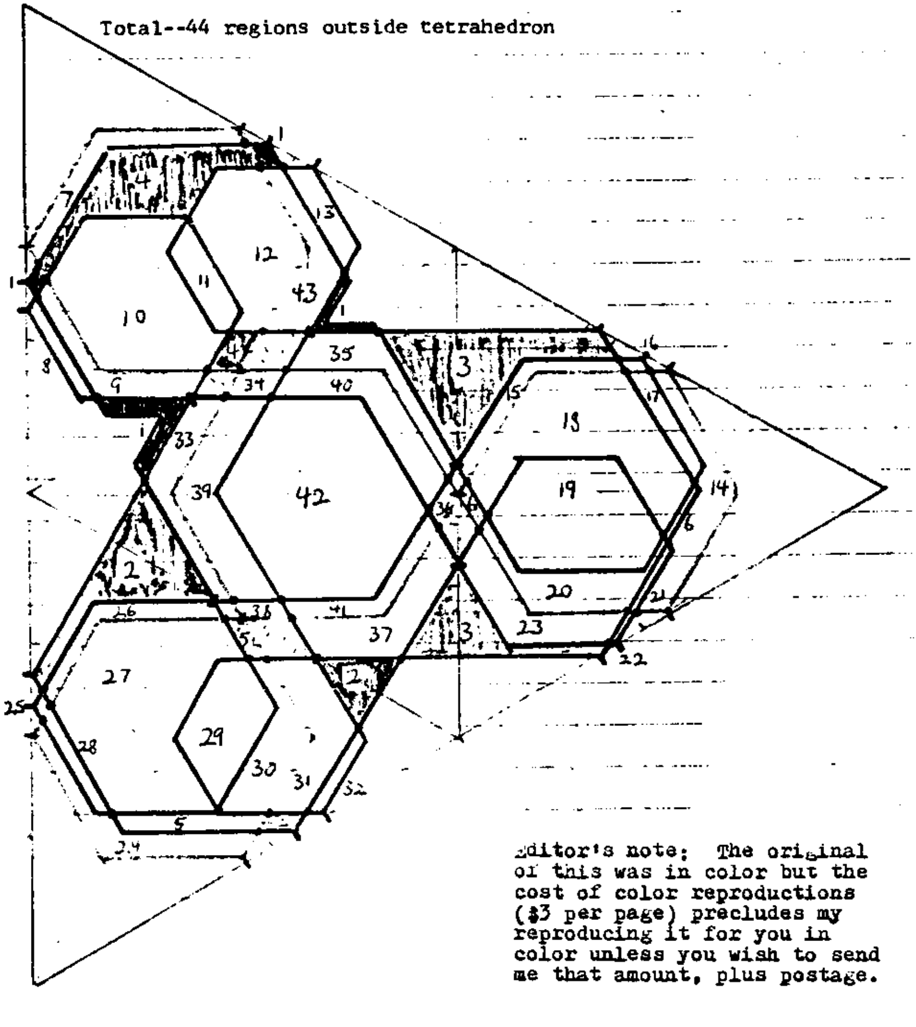




REGIONS OUTSIDE TETRAHEDRON--83 REGION SOLUTION

43 regions sharing one or two surfaces with the tetrahedron,
1 region above, but not touching, tetrahedron's "central" face (above regions 39, 40, 41, and 42)--intersection of red, blue, and green spheres

Total--44 regions outside tetrahedron



Editor's note: The original of this was in color but the cost of color reproductions ($3 per page) precludes my reproducing it for you in color unless you wish to send me that amount, plus postage.

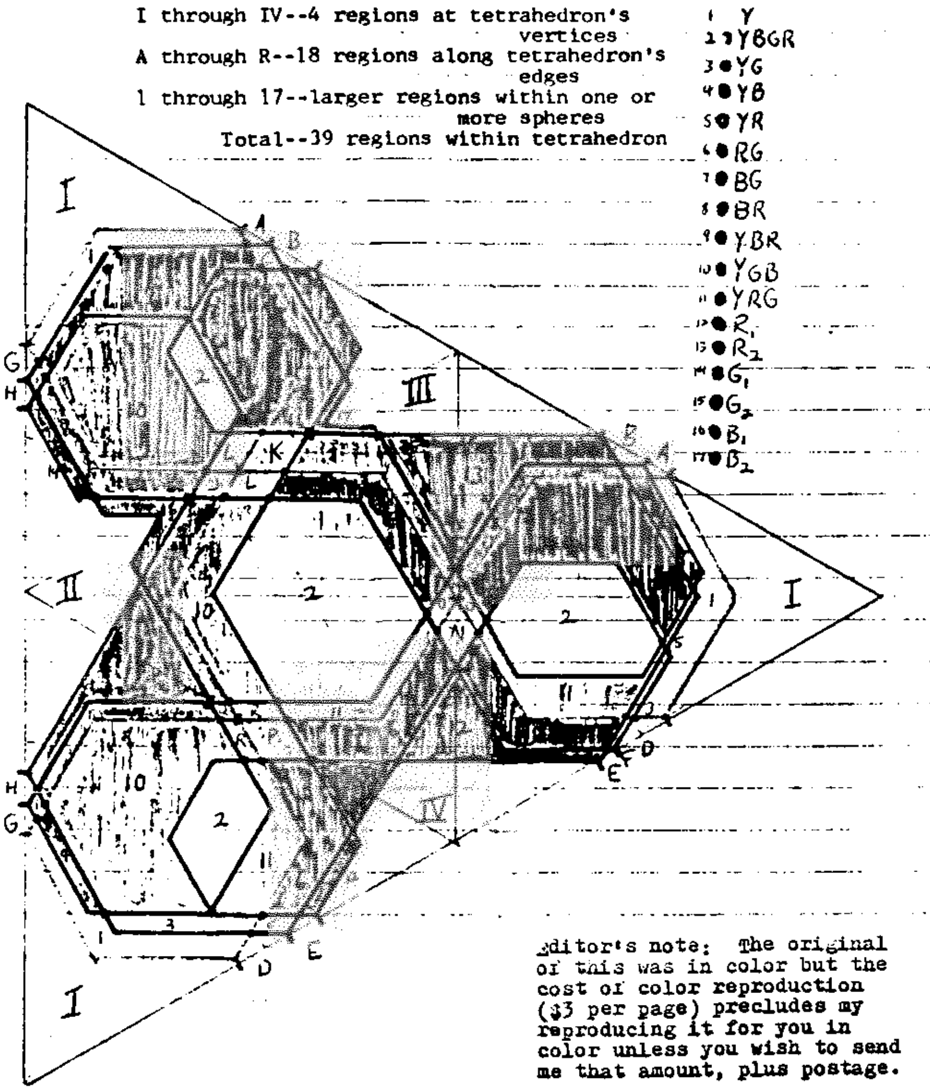
REGIONS INSIDE THE TETRAHEDRON--83 REGION SOLUTION

I through IV--4 regions at tetrahedron's vertices

A through R--18 regions along tetrahedron's edges

1 through 17--larger regions within one or more spheres

Total--39 regions within tetrahedron



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3829 Encino Hills Place
Encino, CA 91436
June 9, 1990

Dear Mr. Hoeflin,

I realize that, in the letter about the four sphere-tetrahedron problem, I wasn't clear about how I arrived at the formula P + 2S - 1 = R. I simply imagined a string of spheres intersecting each other, like beads on a necklace. One sphere contains one region, two spheres contain three regions, strings of three, four, and five spheres contain five, seven, and nine regions, respectively. Among the strings of intersecting spheres, there are no points of intersection, only circles where two spheres intersect. If the first sphere in a chain is swung around to intersect the chain's last sphere and form a ring of intersecting circles, topological simplicity is lost, and the formula doesn't work. I haven't tried to prove the formula or figure out a set of formulas for topological exceptions. I've just gone on the assumption that it is an Euler-type rule that is true for well-behaved interpenetration problems and haven't yet been disappointed by it.

The maximum -- points of intersection among two right cones and a cylinder are as follows: Two cones intersect in a maximum of -- points, and each cone intersects the cylinder in a maximum of -- points. All three shapes intersect each other at -- points, as shown below. The shapes can be oriented so that there are more than -- points at which all three shapes intersect, but only at the expense of points at which only two shapes intersect. The formula is of no help in proving that more than --intersection points among all three shapes always costs more in intersection points between two shapes. This makes me think of the cone-cylinder problem as harder than the sphere-tetrahedron problem.

Thanks again and best wishes,

Richard Sterman

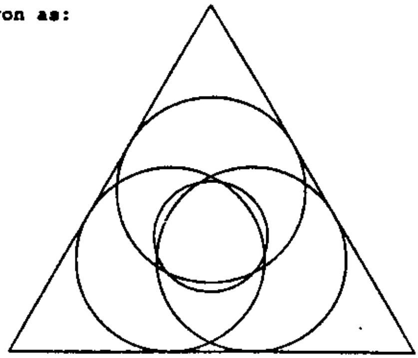
23333 Ridge Route Dr. #51
El Toro, Ca 92630
7 August 1990

Dear Mr. Sterman

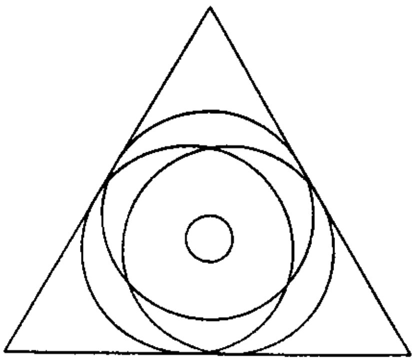
I have seen your letters of 4 and 9 June to Ron Hoeflin regarding the problems of interpenetrating regions.

Your count for the symmetric, one sphere in each vertex solution to the 4 spheres and tetrahedron appears to be too high.

You draw each face of the tetrahedron as:



But in order to make the inner circle intersect each of the other three, its sphere would have to be nestled further into its vertex than any of the other three. If the spheres are displaced by equal amounts, the inner circle should be completely contained in the intersection of the other three.



This gives 4 * 8 = 32 regions outside the tetrahedron, for a total of 57.

Your asymmetric solution certainly gives a lot of regions, but the irregularity makes it much more daunting to count. While I see no apparent error in your count, I feel less than confident in affirming it. (It reminds me too much of some of my earlier attempts at the problem, during which I confused myself so much that I forgot the difficulty with the symmetric solution, and so, over-counted when I rediscovered that situation in another configuration. But I don't recall ever getting anything quite so high. I think I may have missed some of the edge regions.)
Nor did I completely trust the P+2S-1 formula, since it is easy to find exceptions. (disjoint spheres, tangent spheres, concentric spheres) I don't know how to be sure that there are no exceptions in this case, and that there were no miscounts of P as in the symmetric case.

Your cones and cylinder solution agrees with mine, and the formula applied to it agrees with my count.

Best Wishes,

Dean Inada

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First Class
Mail