October 1990 Issue 54

NOESIS

THE MONTHLY JOURNAL OF THE ONE-IN-A-MILLION SOCIETY

Contents

Publisher & Editor

Ronald K. Hoeflin
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Editor's Page

New editor sought: I will probably give up the role of editor of this journal after putting out issue 56 (the December 1990 issue). I have asked Ray Wise if he would like to take over as editor because his recent issue seemed to me to have the sort of variety and intellectual content that I have traditionally tried to include in this journal. I have not yet had a response from him. If anyone else would be interested in the position, let me know. If there are no volunteers, this journal and society will probably be terminated after issue 56. There is not sufficient intellectual or monetary feedback from the members to give me personally sufficient incentive to continue, but the group may have sufficient value to one of you to warrant your investing the time and energy to perpetuate the group. I will send the names and addresses of all current members and subscribers to anyone who agrees to take over as editor. The first person to volunteer will get the job unless I have strong reason to believe you cannot handle it. George Dicks, Jr., for example, has promised to send me material for the journal on several occasions, and has actually claimed to have sent such material, but the promised items (including many back issues of the Mega Society journal that I loaned him) have never materialized, so clearly he lacks the sort of reliability you would want in an editor. C. M. Langan was highly reliable, but his issues were too focused on his own personal interests. Also, he never, to my knowledge, provided any member with his phone number, making him less accessible than would be ideal for an editor. An alternative to perpetuating this society would be for members to join the other one-in-a-million group, the Mega Society, as several of you already have. I am told it published its journal 3 or 4 times a year. If interested, contact its coordinator Jeff Ward, 13155 Wimberly Square, #284, San Diego, CA 92128.

Refunds: Those members who paid $20 for a full year's subscription to this journal (issues 51-62 rather than 51-56) will be sent a $10 refund, which they may then send to the new editor, if we find one. As indicated in issue 52, page 2, these people are as follows: Anthony J. Bruni, Richard May, Richard Sterman, S. Woolsey, and Jeff Wright. Since then, we have also had a $20 membership from Kjeld Hvatum and Marilyn vos Savant. I will gladly substitute a 6-issue subscription (or extension of subscription) to In-Genius in lieu of these refunds. Your subscription to In-Genius would start with issue 25 (January 1991) unless you specified otherwise. All back issues of that journal are still available. It was published monthly from July 1989 to June 1990 and twice monthly since July 1990.

Non-member subscribers: The new editor will have the option of including or excluding non-member subscribers on his or her mailing list. There are currently 14 members and 20 non-member subscribers. I believe the latter ought to be retained, if possible, because they may be able to provide valuable financial as well as intellectual input to the group.

In this issue: I have included in this issue all of the puzzle columns that our member, James Hajicek, did for the Triple Nine Society journal, Vidya. The last two pages of this issue consist of items that appeared in issue 19 of my journal, In-Genius.

Puzzles: # 1

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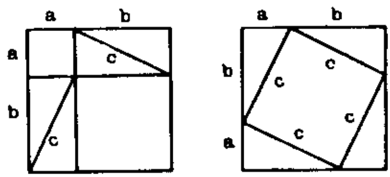
It is my intention to begin a puzzle section, starting with this issue. Readers are encouraged to submit puzzles for publication. Original puzzles are of course preferred, but it is best to have material which is instructive and educational rather than merely a diversion. An ideal puzzle has an edifying principle embedded in its solution.

The solutions to each month's puzzles will be given in the next issue. If possible, readers who send correct solutions to me will be credited.

In lieu of solutions this month, the following geometric proof of the Pythagorean theorem should be of general interest, as it is not too well known. With the existence of a proof this simple, there is no reason for this theorem to be taught in school without a proof being given. For any right triangle with sides a, b, and hypotenuse c, consider the two squares shown here with sides of length a + b. Since these have equal areas, removing the four equal triangles from each must leave equal areas remaining. Thus $a^2 + b^2 = c^2$.



1. As is well known, the integers 3, 4 and 5 satisfy the equation for the sides of a right triangle, as $3^2 + 4^2 = 5^2$. How can one easily generate (without using a computer) many more integer examples of right triangles? Note that we want a variety of interesting solutions, not just simple multiples.

2. The three dimensional analog of the Pythagorean theorem is $a^2 + b^2 + c^2 = d^2$. Here d is the distance between two points separated by distances a, b, and c along three mutually perpendicular axes. As in the first problem, how can one find a variety of integer solutions to this equation?

3. While we are on the subject of triangles, who out there knows (or can find out) the formula for the area of a triangle of arbitrary sides a, b, and c? Most people go through their entire lives without realizing that such a formula exists.

4. It is common to expect answers to problems to be given in the simplest form. For example, fractions should be reduced when possible. The following two expressions have a clumsy square root within a square root. Can they be simplified?

$$\sqrt{5 + \sqrt{24}} \qquad \sqrt{31 - \sqrt{840}}$$

Puzzles: # 2

James Hajicek
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I am sorry about missing the deadline for the last issue. It is difficult to allow time for reader response and still have a monthly column. There were many correct solutions submitted: (4) Donald Vanderpool, Howard Vein, Eugene Primoff, Larry Vikander, Alfred Simpson, Craig Barrington, (3) Ronald Deep, Thomas Burnham, Paul Cook, Name Withheld, (2) Gerald Baker, Rick Magnus, Stephen Jaworowicz, Jack Lawless, (1) Bill Bergdorf, John Pearson, Jere Guin, Charles Ramsey.

Solution 1. To generate integer solutions to $a^2+b^2=c^2$. For any integers m and n, use (a , b , c) = (2mn , m^2-n^2 , m^2+n^2). It can be proven that this will generate all of the relatively prime solutions.

Solution 2. To find integer solutions to $a^2+b^2+c^2=d^2$. A great variety of methods were were received for this, mostly a special case of the formula contributed by Donald Vanderpool (a , b , c , d) = (mp-nq , mq+np , $m^2+n^2-p^2-q^2$, $m^2+n^2+p^2+q^2$). This appears to generate all of the relatively prime solutions. My original idea is also a special case of this with q = 0, which can be proven to generate at least some multiple of every solution. Another interesting idea, based on a contribution by Alfred Simpson, reduces this problem to the first problem with (a , b , c , d) = (px , py , qz , rz) for any p,q,r and x,y,z which satisfy $p^2+q^2=r^2$ and $x^2+y^2=z^2$.

Solution 3. To find the area of a triangle given the lengths of three sides. If the area is expressed in terms of the base and height, the height can be eliminated by using the Pythagorean theorem. Then

$$A = \frac{1}{4}\sqrt{2a^2b^2+2a^2c^2+2b^2c^2-a^4-b^4-c^4}$$

$$= \sqrt{s(s-a)(s-b)(s-c)} \qquad \text{where } s = \tfrac{1}{2}(a+b+c) .$$

The second expression was the one most often contributed by the readers, who tell me that it is known as Heron's formula.

Solution 4. To simplify nested square roots. The following may be verified by squaring both sides of the equations:

$$\sqrt{5+\sqrt{24}} = \sqrt{3}+\sqrt{2} \qquad \sqrt{31-\sqrt{840}} = \sqrt{21}-\sqrt{10} .$$

Problem 1. For what values of f and n is the following description valid? A polyhedron consists of f faces of equal size and shape, each a rhombus with angles A,B,A,B. Meeting at each vertex are either three type A angles or else n type B angles.

Problem 2. We have two flasks. One measures 20 fluid ounces, and the other measures 17. We can fill a flask from a water source, transfer from one flask to the other, or empty down a sink. Without making new marks, how many fills, transfers, and empties are required to measure out 1 ounce? How about 2 ounces?

Puzzles: # 3

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Scores on the two problems from Issue 65 are as follows: (2) Steve Offner, Larry Vikander, Christopher Burch, John Howell, (1) Frederick Miles, Stephen Jaworowicz, Name Withheld. There were also three people who deserve at least half credit on the second problem, solving correctly for 1 ounce, but who fell into the trap of continuing with the same method for 2 ounces.

Solution 1. To find semi-regular polyhedra with rhombus faces. Euler's formula says that any polyhedron with f faces, e edges, and v vertices must satisfy $f - e + v = 2$. Every face has 4 edges shared with two other faces, $e = 4f/2$. Every face has 2 verticies shared with 3 other faces and 2 verticies shared with n other faces, $v = 2f/3 + 2f/n$. Solving, $f = 6n/(6-n)$. Solutions for (n,f) are (3,6), (4,12), and (5,30). The angles of the rhombuses were not requested, but will be of interest to anyone who wishes to build models, so here are my calculated values. Of course $A + B = 180°$. For n = 3, $A = B = 90°$. For n = 4, $\cos B = 1/3$, or $B = 70.53°$. For n = 5, $\cos B = \sqrt{5}/5$, or $B = 63.43°$.

Solution 2. To measure out 1 fluid ounce, using flasks holding 20 and 17. The following chart lists the step, the operation (Fill, Transfer, or Empty), the contents of the 20 ounce flask, and the contents of the 17 ounce flask:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
F	T	E	T	F	T	E	T	F	T	E	T	F	T	E	T	F	T	E	T	F	T	E	T
20	3	3	0	20	6	6	0	20	9	9	0	20	12	12	0	20	15	15	0	20	18	18	1
0	17	0	3	3	17	0	6	6	17	0	9	9	17	0	12	12	17	0	15	15	17	0	17

The same method can be continued onward to measure 2 ounces, but there is a shorter method:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
F	T	F	T	E	T	F	T	E	T	F	T	E	T	F	T	E	T	F	T
0	17	17	20	0	14	14	20	0	11	11	20	0	8	8	20	0	5	5	20
17	0	17	14	14	0	17	11	11	0	17	8	8	0	17	5	5	0	17	2

Problem 1. This was suggested by another member. A man starts from the point where the prime meridian crosses the equator, and walks 45° northeast, constantly correcting his course with a geographic compass which always points toward the geographic pole. He walks with equal facility on land, water, and ice. Mountains do not deflect him from his course. Does he reach an end point? If so, where, and how far will he have travelled when he gets there?

Problem 2. This was given to me in 1980 by a co-worker. In a doubles match tournament 16 players each play 15 games with a different partner each time. Each player has each other player as an opponent exactly two times. Find a suitable pairing arrangément for the 15 rounds of 4 games per round.

Solutions to these problems will be given in Issue 69, but don't take too long to submit your contribution.

Puzzles: # 4

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Scores on the two problems from Issue 67 are as follows: (2) Tom Davidson, Raymond Burhoe, (1) Walter Kopp, Craig Barrington, Larry Vikander, Steve Offner. New problems will be given in the next issue.

Solution 1. To find the length of a curve, starting at the equator, with a constant 45° heading. Walter Kopp informs us that this is called a loxodromic curve. See also "rhumb line" in your dictionary. The curve spirals around the north pole, making an infinite number of loops as it approaches the pole. However, the total length of the curve is finite. Without attempting to construct the exact curve, the following argument is sufficient to obtain the length L. For each distance traveled, d, the pole is approached by an amount equal to d cos45°. This is true for infinitesimal distances at all points, and is therefore true in general. Thus $C/4 = L \cos 45°$, where C is the circumference of the earth and $\cos 45° = \sqrt{2}/2$. Finally, $L = \sqrt{2}\,C/4$.

Solution 2. To find a pairing arrangement for a doubles match tournament for 16 players, so that every player has a different partner each time, and so that every player has every other player as an opponent twice. My solution is more complex, but three people submitted some variation of the method given here. The players are identified with letters from A to P. The 16 players are then split into 4 groups, 5 different ways, such that no two groups have two players in common. Each group then plays three rounds among themselves, pairing in different ways.

	Split 1	Split 2	Split 3	Split 4	Split 5
Group 1	A B C D	A E I M	A F K P	A G L N	A H J O
Group 2	E F G H	B F J N	B E L O	B H K M	B G I P
Group 3	I J K L	C G K O	C H I N	C E J P	C F L M
Group 4	M N O P	D H L P	D G J M	D F I O	D E K N

Round	Game 1	Game 2	Game 3	Game 4
1	A B - C D	E F - G H	I J - K L	M N - O P
2	A C - B D	E G - F H	I K - J L	M O - N P
3	A D - B C	E H - F G	I L - J K	M P - N O
4	A E - I M	B F - J N	C G - K O	D H - L P
5	A I - E M	B J - F N	C K - G O	D L - H P
6	A M - E I	B N - F J	C O - G K	D P - H L
7	A F - K P	B E - L O	C H - I N	D G - J M
8	A K - F P	B L - E O	C I - H N	D J - G M
9	A P - F K	B O - E L	C N - H I	D M - G J
10	A G - L N	B H - K M	C E - J P	D F - I O
11	A L - G N	B K - H M	C J - E P	D I - F O
12	A N - G L	B M - H K	C P - E J	D O - F I
13	A H - J O	B G - I P	C F - L M	D E - K N
14	A J - H O	B I - G P	C L - F M	D K - E N
15	A O - H J	B P - G I	C M - F L	D N - E K

Puzzles: # 5

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The problems given here this month are intended to illustrate some of the current research in artificial intelligence. Solutions will be given next month, with no time for reader response.

Problem 1. The following logic problem was presented in 1978 by L. Schubert of the University of Alberta as a challenge for automated reasoning computer programs. It became known as "Schubert's Steamroller" when it was found to be too hard for the existing theorem provers because it has a large search space (when the assumptions are variously combined in an uninspired way). It resisted automatic solution until 1984, when an automatic theorem prover was constructed using more advanced principles. See Walther, C., "A Mechanical Solution of Shubert's Steamroller by Many-Sorted Resolution," Proceedings of the National Conference on Artificial Intelligence, August 1984.

Wolves, foxes, birds, caterpillars, and snails are animals, and there are some of each of them. Also there are some grains, and grains are plants. Every animal either likes to eat all plants or all animals much smaller than itself that like to eat some plants. Caterpillars and snails are much smaller than birds, which are much smaller than foxes, which in turn are much smaller than wolves. Wolves do not like to eat foxes or grains, while birds like to eat caterpillars but not snails. Caterpillars and snails like to eat some plants. Prove that there is an animal that likes to eat a grain -eating animal.

In order to make this a puzzle, let us ask, "What animal is it that likes to eat what grain-eating animal?" Although I trust that our members will not find this problem overly difficult, it illustrates the current level of machine intelligence for problems of this type.

Problem 2. Logic circuit design is replete with problems, some practical, and some (like this one) merely amusing. Here is one taken from the book Automated Reasoning, Introduction and Applications (Prentice-Hall, 1984) by Wos, Overbeek, Lusk, and Boyle. This book is filled with examples of how various puzzles and problems are formulated in symbolic logic for computer solution. It is an excellent introduction to this topic, especially if one has some previous knowledge of symbolic logic.

Logic signals are carried on wires, and are either a zero or a one. An "inverter" has one input and one output, the output being the opposite of the input. An "and gate" has two inputs and one output, the output being a one if both of the inputs are one, and a zero otherwise. An "or gate" has two inputs and one output, the output being a one if either or both of the inputs are a one, and a zero otherwise. The problem is to design a circuit with three inputs and three outputs, each output being the opposite of the corresponding input. Clearly this can be easily done with three "inverters," but the problem is to do it using only two "inverters" plus a minimum of "and gates" and "or gates."

Puzzles: # 6

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Here are the solutions to the problems in Issue 70. New problems will be given in the next issue.

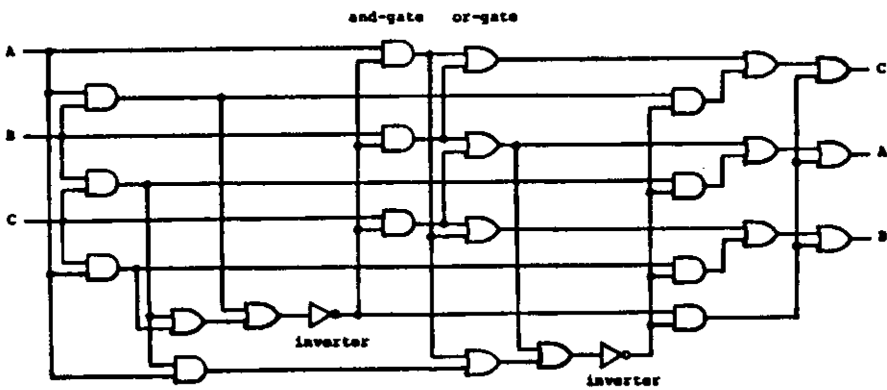
Solution 1. To find what animal likes to eat what grain-eating animal. The key assumption is "every animal either likes to eat all plants or all animals much smaller than itself which like to eat some plants." This is used three successive times in the following abbreviated proof.

There are some snails, and snails eat some plants, and snails are much smaller than birds, but birds do not like to eat snails. Therefore, birds like to eat all plants. There are some grains, and grains are plants. Therefore, birds are a grain-eating animal.

Wolves do not like to eat grains, and there are some grains, and grains are plants. Therefore, wolves like to eat all animals much smaller than themselves which eat some plants. Wolves do not like to eat foxes, which are much smaller than wolves. Therefore, foxes eat no plants.

Foxes eat no plants, and there are some plants (grains). Therefore, foxes like to eat all animals much smaller than itself which like to eat some plants. Birds are much smaller than foxes, and eat some plants (grains). Therefore, foxes eat birds.

Solution 2. To invert three signal lines using only two inverters, plus a minimum of additional or-gates and and-gates. The key to solving this problem is in determining which two signals should be inverted. They are: the signal which says that two or three of the inputs are a one, and the signal which says that one or three of the inputs are a one. My best effort on this problem used 26 additional gates. The computer-assisted solution, from the book Automated Reasoning, requires only 25. While studying this, I discovered (to my dismay) a simplification which requires only 24, as follows:



Puzzles: # 7

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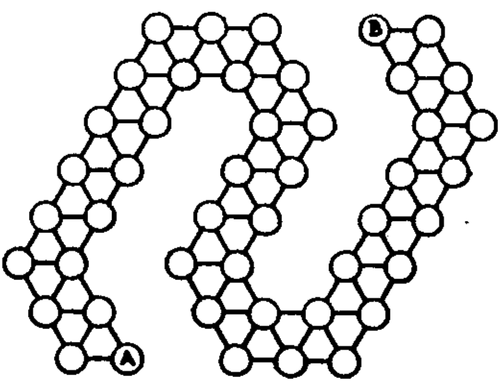
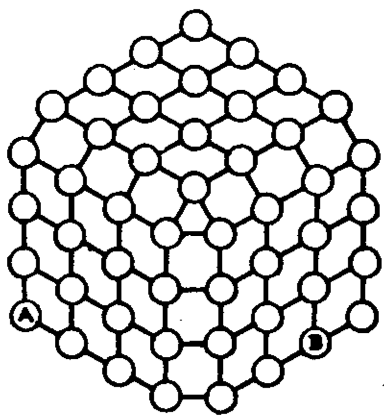
There is a close connection between puzzles and games (that is, two person games with complete information), because every game position is a puzzle to determine the best move. Playing a game is like solving a sequence of puzzles. Equally, every game position is a puzzle to determine which player (if any) can force a win.

In the following two game positions we would like to know which player, A or B or neither, can forcefully win the game. Also, there are two alternatives to be considered: assuming that player A has the first move, and assuming that player B has the first move. Our readers are invited to submit the solutions to me, and I will give credit for correct solutions in a following issue.

The rules of the two games are the same. I call either game "The Web." The two players each have one piece, located at some vertex of the web. The players take turns moving their piece one step in any direction along the lines of the board. The game is won when one player captures his opponent's piece. The opponent's piece may be captured when it is next to the player's piece, so that the player can move one step into the spot occupied by the opponent, removing the opponent's piece from the board.





Cartoon

Bill Irvin
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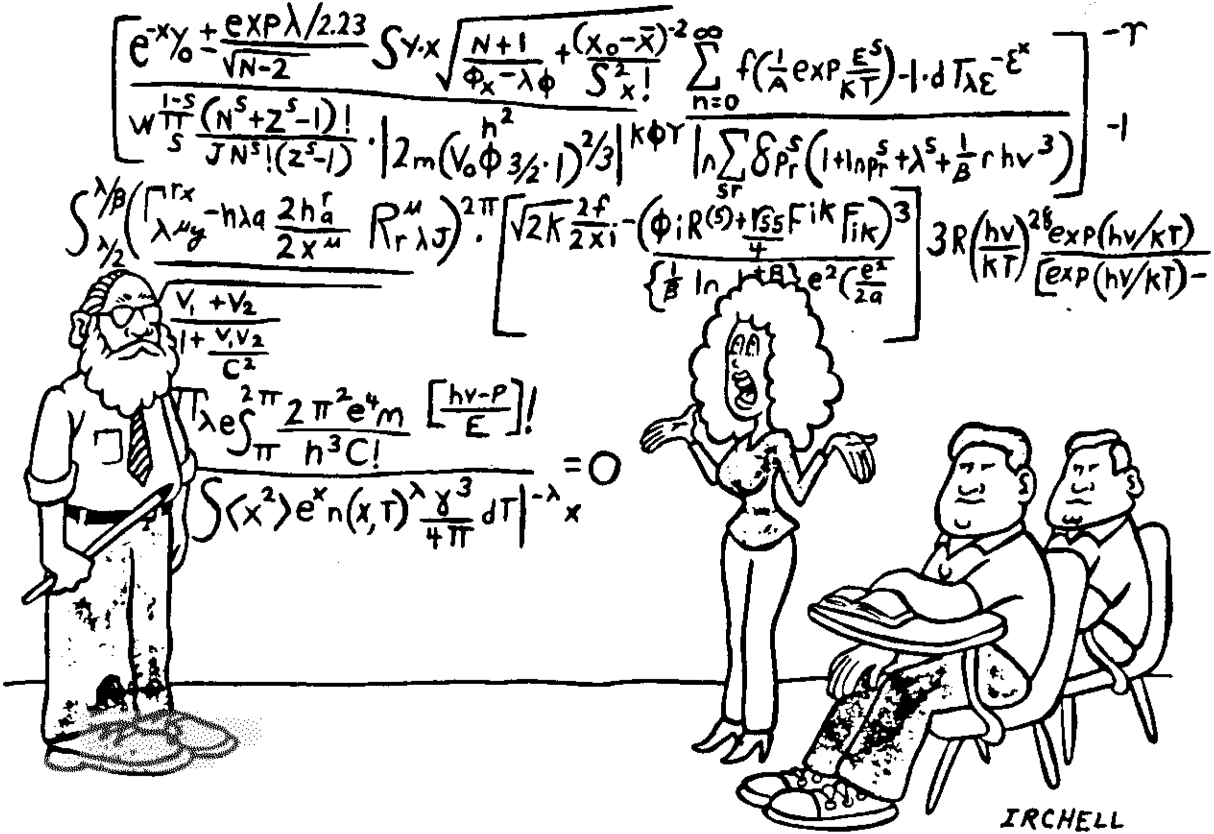

"It just seems like an awful lot to go through to end up with nothing."

September 10, 1990

Grace McWhorter LeMonds
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Lubbock, Texas 79424
806 794-4600

Ronald K. Hoeflin, Ph.D.
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Dear Dr. Hoeflin:

Thank you for the very interesting packet of mail I received today as an editor of a Mensa publication. I will reply to that information at a later date.

In the meantime, as an Educational Diagnostician in the State of Texas (psychometrician, in other states), may I correct you on one vital point? The ultimate score on the Stanford Binet Test of Intelligence is 136, using the 1972 norms. It was 147 on the 1960 norms.

Unless there are new norms of which I am unaware, 136 is the only score possible within the 99th percentile. 132 through 135 delineate the 98th percentile ranking.

Sincerely yours,

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