

Noesis

The Journal of the Mega Society Issue 88 December 1993

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MEMBERSHIP ROSTER
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As promised, here is the list of current and recently lapsed Mega Society members: 3/4 of these have at least one qualifying score on a test written by Ron Hoeflin. Another half-dozen (denoted by an "A" after their name) qualified by another instrument. Two people (denoted by a "C" after their name) are subscribers who probably could qualify but have not yet submitted their Mega or Titan Test score sheet. Ron Hoeflin does not consider himself a member, but everyone else does. Please contact me if there are errors in this list.

Philip Bloom (A)
Geraldine Brady
Anthony J. Bruni
Chris Cole
Robert Dick (A)
George Dicks
Eric Erlandson
Marcel Feenstra
James D. Hajicek
Chris Harding (A)
Ron Hoeflin
Kjeld Hvatum
Dean Inada
C. M. Langan
Kevin Langdon (A)
Richard May
Glenn A. Morrison

Johan Oldhoff
A. Palmer
Dr. P. A. Pomfrit
Carl Porchey (A)
M. C. Price
Keith Raniere
Rick Rosner
Kevin Schwartz (C)
Steve Sweeney (C)
Jeff Ward
S. Woolsey (A)
Jeff Wright

A REPLY BY CHRIS COLE

Bob Hannon (issue 87) asks "who asked Chris to respond to my articles published in (85)." Well, I can only say that when Bob sent me a letter personally and asked me to respond, I interpreted that as a request to respond. Clearly I misunderstood. In the same article, Bob goes on to accuse me of threatening to censor his articles. I did not do that. I said that if Bob wants me to respond to his articles, I will do so only if he keeps them down to two pages per issue. So far, we have published everything Bob has submitted (indeed, the January issue will be devoted entirely to Bob). However, I don't think it's healthy for *Noesis* to contain so much material from one author. Also, I think Bob wants people to respond to his ideas. So again, I offer to respond, but only to one two-page article per issue.

A Single-Pass Algorithm for Calculating the Variance of a Sample

Marcel Feenstra

In the well-known formula for the variance of a sample,

$$\text{Var} = \frac{1}{N-1} \sum (X_i - \bar{X})^2$$

\bar{X} is the mean of the total sample, not some running average. It seems, therefore, as if we first have to calculate the mean of the sample before we can calculate its variance. (Note that, if we use this "natural" two-pass algorithm to calculate the variance of a sample, we need the individual observations not only during the first pass but also during the second pass, so we have to save them.)

However, instead of the formula above we can write equivalently:

$$\text{Var} = \frac{1}{N-1} \sum (X_i^2 - 2X_i\bar{X} + \bar{X}^2)$$

or:

$$\text{Var} = \frac{1}{N-1} (\sum X_i^2 - 2\bar{X}\sum X_i + N\bar{X}^2)$$

Therefore we can also determine the variance of a sample by keeping track of 1) the number of observations N , 2) the sum of the squares of the individual observations and 3) the sum of the individual observations. (The mean and its square can, of course, be calculated at all times using N and the sum of the observations.) This one-pass algorithm does not require saving the individual observations; it could be used to continuously determine the "running variance" of an "ongoing stream of observations".

Note: In a computer implementation, the one-pass algorithm is more sensitive to overflow than the two-pass algorithm (when there are many observations or the absolute value of the observations is relatively large); however, it is less sensitive to underflow (when the difference between individual observations and the mean of the sample is very small).

* * * * *

This algorithm occurred to me recently; is it original, or has anyone seen it before?

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early December, 1993

Chris Cole
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Dear C & R --

An up-to-date Mega address list sounds "way", but not be "way-er" and go for an ISPE-like Roster, in which affiliates can list additional information such as age, vocation, special skills, special interests, general comments, whatever? That way, we can all get to know one another and be more like a society rather than just recipients of a newsletter.

Someone -- Rick? -- asked: would Megarians have been recruited for the Manhattan Project, had it transpired c. 1992-95 rather than c. 1942-45? Leaving aside S. Golomb & H. Taylor (neither of whom, so far as I know, ever joined any HiQ clubs); I seriously doubt it. Not to sound rude; but mainstream science texts don't even footnotes past or present Megarians. The general scientific community doesn't give a hoot about the Mega Society. Should it?

Presumably Noesis readers followed the media frenzy re Princeton's Professor Andrew Wiles and his reputedly "out-of-the-blue" FLT proof. According to the Official Story, Wiles and pal John Conway -- JHC is one of the great geniuses of our century -- duped their dopey colleagues until the end of that three-day lecture in England. Yet note a peculiar foreshadowing in Noesis 81 ("May"), p. 6: "Don't you (Conway) have more constructive things to do? Proving Fermat's Last Theorem or something?" It's almost as if... as if "KvJ" suspected something...

In truth, my freshman year my friends and I knew it was just a matter of time before someone at Princeton cracked FLT. Most of us had our money on Gerd Faltings, who'd just shocked the math world by proving the Mordell Conjecture. Several of my friends were busy with their OWN attempts at proofs -- and maybe if they'd just had another 350 years to work it out, one or more of them would've succeeded.

Have all you read Parade's "Ask Marilyn" column regarding FLT? To me, her "proof" that FLT remains unproved seems specious. Marilyn's argument runs something like this: if Euclidean construction problems -- such as doubling the volume of a cube -- solvable only in non-Euclidean geometry, don't count as true solutions, then Wiles' proof shouldn't count either, since it too relies on non-Euclidean geometry.

Plato used this kind of brilliant *a priori* sophistry throughout his works. (Given: A produces B. Given: C produces D. QED: AC produces BD.) With it, one can "prove" anything, from black is white, to God exists, to slavery is a moral necessity.

I don't know who, if anyone, Marilyn consulted before she published her essay, but personally, I'd never bet against Conway in math. If he said, "2 and 2 makes 5", I'd say: "Okay; show me why." And Conway's not the only stooge, by Marilyn's reasoning.

Here, as best I can see it, is the fatal flaw in Marilyn's solecism. A Euclidean construction problem, to remain such, by definition, must be solved within Euclidean geometry. Moreover, vector points just one way: anyone can "construct a trisected angle"

otherwise -- construct an angle; then copy it; then copy it again... *voila* -- a trisected angle. Cripes. In junior high, when you goofed off in math class, didn't your teachers assign you those infuriating "proofs" of the Parallel Postulate in which you to find the subtle flaw -- which invariably was that at least one of the logical steps ultimately traces back to the presumption of the truth of the Parallel Postulate?

True, FLT is related to the Pythagorean Theorem, which is Euclidean. Yet FLT itself is not geometric at all -- it's pure number theory. Its truth or falsehood no more relies upon the geometry in which one proves it than in relies upon the color of the paper on which it's printed. So what's the deal? Am I missing a step here? If so, can someone please explain to me what it is?

Rick, how's the move going? Maybe because I've had more residences than birthday candles, or maybe because I'm an in-the-rut kinda guy -- I despise everything related to moving. I hate wrapping newspapers around glasses and stuffing them into crates, ink gradually blackening your hands and clothes. I hate filling and carting book boxes until they all look the same. I hate squeaky Styrofoam "packing peanuts". I hate dragging couches and tables up and down stairs. I hate travelling, whether by car, train, plane, boat, or bus. I hate house-touring. I hate sleeping in cobwebbed bedrooms. I hate scraping lead paint off walls. Yuck, yuck, yuck.

In a Carrollian fit ("frit"?), more Pom-possibilities occurred to me.

- P) 2349 (duh, how did I miss THAT one?)
- JJ) 5 4 3 6....

Comment: I'd assumed, since there were no commas, this sequence HAD to be some decimal expansion. The problem, as I noted, was there were too many mildly plausible numbers, and no obvious favorites. Simpler explanation: each numeral represents the number of letters used to spell out the (sequential) counting numbers.

- H) 78 ???

Comment: A shot in the dark. As George -- of The Magnificent Ambersons -- might have said, "Here's a queer duck!" If I recall, each number in the sequence is precisely 1 letter longer than the preceding number. Such a sequence could presumably go on forever, since, for instance, "one hundred one" is exactly 1 letter longer than "seventy-eight"; and since you can keep building at either end. -- Yet I'm missing something critical here; the numbers listed are the 1st numbers of N letters neither alphabetically nor numerically! I should to keep thinking about this one; but in all honesty, I've kinda way-overdosed on number sequences. Let's have some more A. Morrison word puzzles!

More pesky problems:

From an ordinary deck (52 cards), what's the random chance of drawing a hand (five cards) with at least three primes (aces are high)?

Imagine an ordinary cubic die, but coated with glue. To each face you fasten the face of an identical die (minus the glue). You wind up with a sorta 3-D "X", with spots on

all thirty faces. Counting rotations as identical, but mirror images as separate, how many such distinct objects are possible.

Imagine the same thing, but a cube of 2^3 dice. Of 3^3 dice. Etc. Come up with a general formula -- and make it snappy!

How many different ways can you fold an array of 5 by 4 stamps into a twenty-"page" booklet? What about N by M stamps?

Imagine a 4-D hypersphere surrounded on all "sides" by identical hyperspheres. How many, minimum, are sufficient so no straight line can be drawn from the central hypersphere which doesn't pass through the obscuring hyperspheres on either side? I suspect 5, 500 will more than suffice, and that 4, 000 will not suffice; but that's a guess.

My earlier "approximation answer" for Cole's marble problem made "unfair" use of what Keynes termed the Principle of Indifference. (This is the principle which screwed up so many people on the three-door problem.) An interesting question is: how useful IS this Principle when it is "unfairly" used?

In a column, Martin Gardner writes of a bet where you try to guess the average size of boxes created by a machine which "randomly" makes boxes between size A and B. You can see boxes A and B; but you don't know if the size is determined by length or by volume (length^3). According to Gardner, the Principle of Indifference is therefore inapplicable.

Admittedly, if the gamut runs from 1 to 10, 000, you must guess one way or the other. But what if the gamut runs from 500 to 505? You may not know the method of randomization, but you can guess a good ballpark figure either way.

Does anyone have the mathematical background to explore the issue of using and abusing the Principle of Indifference? This might make for a cool Sci Am article.

LeRoy Kottke kindly sent me a note regarding my issue-81 comments on his issue-80 physics problem-set. I'm as energetic as a hypothermic three-toed sloth; since I figured he wanted a reply before the next year (AD) which square-roots without a remainder, in the margins (which were too small to contain a most amazing proof... never mind) of a photocopy of his letter I hurriedly scribbled my comments on his comments. LRK: no offence or disrespect intended.

Hi again to Bob, Frank, & Arthur; I'll write to you as soon as I can think of anything interesting to say.

Sholomly yours,

Kevin L. Schwartz

THE EINSTEIN-LORENTZ TRANSFORMATION

Robert J. Hannon
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31 Aug 93

in his seminal paper on the subject now known as the Theory of Special Relativity ("On the Electrodynamics of Moving Bodies", Annalen der Physik, 17, 1905) Einstein derived a set of simultaneous equations, now known as the Lorentz Transformation. These equations were intended to relate the spatial and temporal standards of measurement (the metrics) of two frames of reference in constant, linear, relative motion in homogeneous, empty, field-free, space and time. Frames of reference meeting these conditions are Inertial Frames of Reference (IFRs).

A) Einstein's derivation was predicated on the postulate that the velocity of propagation of electromagnetic radiation, C , is the same in all IFRs. It was also based on measuring the coordinates (x and t) of a point in the metrics of IFR-K relative to its origin ($x=0$ and $t=0$), and transforming the position of that point to the corresponding coordinates (x^* and t^*) in the metrics of IFR-K*.

Einstein's algebraic procedures involved the instantaneous distance between the origins of the two moving IFRs. V , the relative speed of the two IFRs in the direction parallel to the x and x^* coordinates, is the same in the metrics of both IFRs. The instantaneous distance (in the direction parallel to x^* and x) between the origins, according to Einstein, is Vt . The instantaneous time interval (in the direction parallel to t^* and t) between the origins, according to Einstein, is Vx/C^2 .

One of the results of Einstein's procedure is to make x^* a function of x and of both V and t . A similar situation arises in his derivation of the transformation of times: t^* is a function of t and both V and x .

B) Einstein's procedure produced the Lorentz Transformation:

$$(B-1) \quad x^* = (x - Vt)\tau$$

and

$$(B-2) \quad t^* = (t - Vx/C^2)\tau$$

$$\text{where:} \quad \tau = 1/\sqrt{1 - V^2/C^2} = 1/\sqrt{1 - \beta^2}$$

(B-1) and (B-2) are the entire mathematical foundation of Einstein's Theory of Special Relativity.

(B-1) and (B-2) relate ("transform") the metrics (x and t) of IFR-K to the metrics (x^* and t^*) of IFR-K*. When $V=0$, $x^* = x$, and $t^* = t$; if $x = 1$ meter, then $x^* = 1$ meter; if $t = 1$ second, then $t^* = 1$ second. As measured by an observer resident in IFR-K, the metrics x and t do not change as V is changed. As measured by an

observer resident in IFR-K*, the metrics x* and t* do not change as V is changed. However, as measured by an observer in IFR-K*, metrics x and t may appear to change relative to x* and t*, as V is changed. Similarly, as measured by an observer in IFR-K, metrics x* and t* may appear to change relative to x and t, as V is changed.

C) Einstein sought to determine the relationships between x* and x, and between t* and t, as V is varied. Mathematically, he sought to determine two functions of V: F(V) and f(V):

$$(C-1) \quad x^* = x[F(V)]$$

and

$$(C-2) \quad t^* = t[f(V)]$$

under the postulate that C is the same in all IFRs. The simplest mathematical form of that postulate is:

$$(C-3) \quad x/t = C \quad \text{or} \quad x = Ct$$

and

$$(C-4) \quad x^*/t^* = C \quad \text{or} \quad x^* = Ct^*$$

Given (C-3) and (C-4) the Law of Equivalences mandates:

$$(C-5) \quad x/t = C = x^*/t^*$$

which means that x/t and x*/t* are always equal, regardless of the value of C. (C-5) neither contains nor implies relationships between t and x*/t* and any other parameter, such as V, and the equality of those two ratios is independent of all other variables. (C-5) makes all attempts to find a "transformation" between x/t and x*/t* futile and unnecessary: (C-5) is that transformation. Nevertheless, (C-5) does not require that x* = x, nor that t* = t; it requires that the ratio x/t equal the ratio x*/t*, and that both ratios equal C. (C-5) also requires that x*/x = t*/t.

The Einstein-Lorentz Transformation (B-1) and (B-2) may be applied only in accord with (C-5): x/t and x*/t* may not have values other than C.

Physically, x/t is a velocity relative to, and measured in the metrics of, IFR-K; and x*/t* is a velocity relative to, and measured in the metrics of, IFR-K*.

The relationship between V and C is provided by the fact that, since both are velocities, their instantaneous relationship is their ratio:

$$(C-6) \quad V/C = \beta$$

(C-6) also means that $V = \beta x/t = \beta x^*/t^*$.

D) Einstein stipulated that the transformations which relate x* to x, and t* to t, must be linear, because he assumed that IFR-K and IFR-K* exist and move in homogeneous space and time.

(continued in Noesis 89)

EIT 1.2 (Eight Item Test)

I have designed the following problems to study high intelligence. Each of the following eight problems has a correct solution. The correct solutions are relatively straightforward and, once perceived, will usually leave little doubt regarding their correctness. I have not attempted to mislead the test taker or to "hide" higher-level problems beyond what is specified in the instructions (e.g., the letters of the answers have not been selected so as to spell out a sentence). However, these problems have been designed to be extremely challenging. I expect that very few people will be able to find more than one or two correct solutions. If you are seriously interested in finding the correct solutions, do not give up on a problem until you have spent at least several hours trying to solve it. (Then again, the correct solution may just "jump out" at you.)

For each of the eight problems, find an operation that when applied to figure 1 yields figure 2 and when applied to figure 3 yields one of the first eight lettered figures ('a' through 'h'). Once you have found such a *viable* operation (and there should be very few), apply that operation to figure 5 and figure 7. If you thus obtain figures that are among the sixteen lettered figures ('a' through 'p'), you have a *valid* operation. If your viable explanation is not valid, repeat the process. Finally, if your valid explanation is too complex or inelegant, repeat the process. When you find the *best*, valid operation (the simplest and most elegant, valid operation), report your answer by writing the problem number followed by three letters corresponding, respectively, to figures 4, 6, and 8. Remember that the first letter is restricted to 'a' through 'h'. Therefore, "6mna" is *not* a valid answer. Repeated letters (e.g., "1aaa") are allowed. Aax — Box 1391 — Princeton, NJ 08542

(Instructions continue on page 2.)

<p>1</p> <p>1 2</p>		<p>3 4</p>		<p>a b c d e f g h</p>							
<p>5 6</p>		<p>7 8</p>		<p>i j k l m n o p</p>							
<p>2</p> <p>1 2</p>		<p>3 4</p>		<p>a b c d e f g h</p>							
<p>5 6</p>		<p>7 8</p>		<p>i j k l m n o p</p>							
<p>3</p> <p>1 2</p>		<p>3 4</p>		<p>a b c d e f g h</p>							
<p>5 6</p>		<p>7 8</p>		<p>i j k l m n o p</p>							
<p>4</p> <p>1 2</p>		<p>3 4</p>		<p>a b c d e f g h</p>							
<p>5 6</p>		<p>7 8</p>		<p>i j k l m n o p</p>							

EIT 1.2 (Eight Item Test)

When working on the problems, keep in mind the following points: (1) All figures are drawn to scale with high accuracy (as an extreme example, notice that in figure 3j the two squares are *not* of the same size, a fact that is easily perceived by noting that all triangles formed by their intersections are not equal). (2) All lines should be considered perfectly straight and to have no thickness. (3) Many of the figures can be obtained by drawing lines between intersections in a 4 by 4 grid; the others require a larger grid (32 by 32 will suffice for all of them). (4) Sizes, relative proportions, and relative positions matter. (5) A good explanation should require 4 or fewer "steps" (every step is a simple operation applied identically to one or several parts of a figure; e.g., rotate 45 degrees). (6) Do not settle for a valid explanation; find the *best* one.

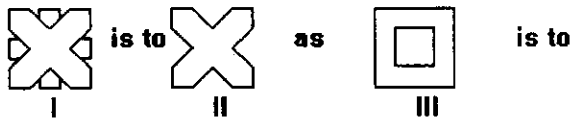
To practice working on problems of this type, I highly recommend that you first try SSFIT (Self-Scoring Four Item Test), a *much* easier test. You can obtain it by sending me a SASE and \$1. To obtain your EIT score, mail me a sheet containing your answers and as much as possible of the following data: full test name (i.e., "EIT 1.2"), name, address, age, sex, SAT scores, GRE scores, and scores on IQ tests. Also, please provide three lists ranking the problems by: (a) how difficult you found them, (b) how much you like them, and (c) how satisfied you are with your answer. Finally, please indicate how much time you spent on each item. (Your score will be based solely on your selection of figures.) Explanations are not required, but could help me to detect errors in the test. I will appreciate a donation of \$10, as it will help to cover the costs of test distribution.

You can copy and distribute EIT as long as you include the copyright notice and do not charge for it. Aax — Box 1391 — Princeton, NJ 08542

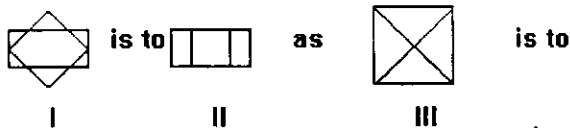
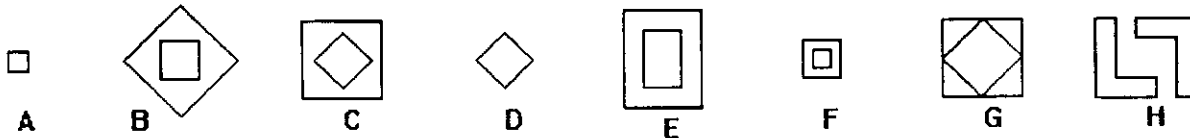
5		3		a									
					?								
1	2	3	4	a	b	c	d	e	f	g	h		
5		7		i									
		?			?								
5	6	7	8	i	j	k	l	m	n	o	p		
6		3		a									
				?									
1	2	3	4	a	b	c	d	e	f	g	h		
5		7		i									
		?			?								
5	6	7	8	i	j	k	l	m	n	o	p		
7		3		a									
				?									
1	2	3	4	a	b	c	d	e	f	g	h		
5		7		i									
		?			?								
5	6	7	8	i	j	k	l	m	n	o	p		
8		3		a									
				?									
1	2	3	4	a	b	c	d	e	f	g	h		
5		7		i									
		?			?								
5	6	7	8	i	j	k	l	m	n	o	p		

SSFIT 1.3 (Self-Scoring Four-Item Test)

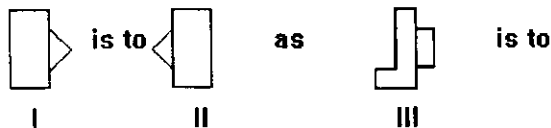
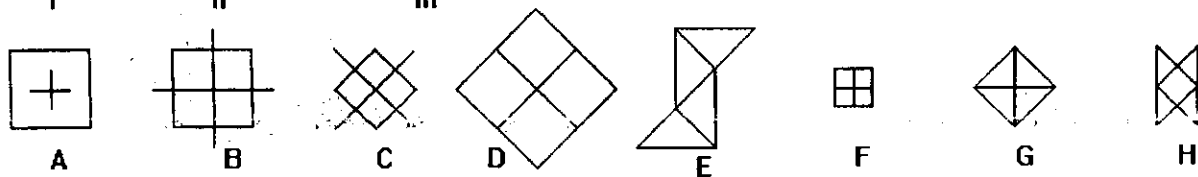
I have invested a considerable effort in creating this test. I am distributing it free of charge for your enjoyment. You can make and distribute as many copies as you want as long as you do not modify the contents in any form, include the copyright notice, and do not charge more than 20 cents per copy. If you want to test an individual or a group, you may retain the other side of this sheet until the testee (or testees) has (have) finished the test. Once the individual or the individuals has or have finished the test, you must give her, him, or them the other side of this sheet. In return for my efforts I am asking that all scores are reported to me. In addition to the score I would like as much as possible of the following information: selected choices, detailed problem explanations, age, sex, address, SAT math score, SAT verbal score, and score on IQ tests. If you are interested, you can obtain another test, the EIT (Eight Item Test), by sending me a self-addressed, stamped envelope and \$2. This other test, which you may have received simultaneously with this one, is *much* more difficult and is not self-scoring. Write to me at: P. O. Box 1391, Princeton, NJ 08542-1391



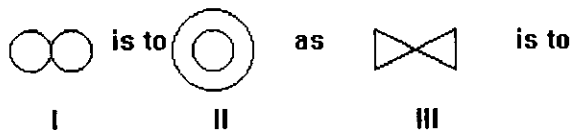
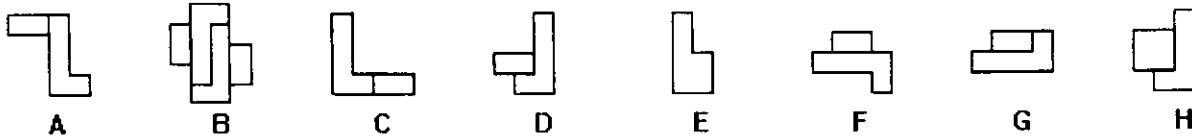
Problem 1



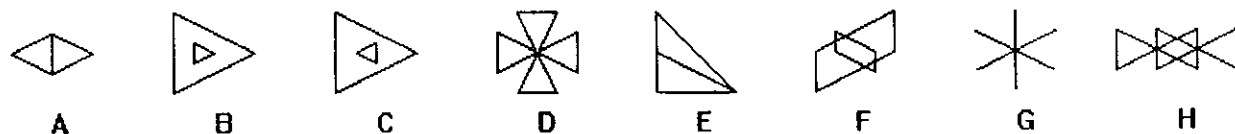
Problem 2



Problem 3



Problem 4



Instructions for SSFIT 1.3

- For each problem, find an operation that when applied to object I yields object II, and when applied to object III yields one of the objects A through H.
- For each problem, try to make your operation as well defined and as detailed as possible.
- Carefully verify that your explanations work perfectly. They should account for every detail of each figure.
- For each problem, if you see several operations that work, choose the one among them that is the simplest and the most precise.
- Write down your choices for all four problems and your explanations in as much detail as possible.
- Follow the procedure described below to find out how many of your choices (not your explanations) are correct. If any of your choices is not correct, go back to step 1. If you do not cheat, the procedure should allow you to find out how many of your choices are correct without telling you which are the correct choices. Do this step only once.
- Write down your final choices for all four problems and your explanations in as much detail as possible.
- Make sure it is clear what your choices are, and make sure your explanations are detailed enough.
- Read the solutions to the problems.
- Score 10 for each correct answer with a perfect explanation.
- Score 8 for each correct answer with a strong explanation that is missing a small detail.
- Score 6 for each correct answer with an explanation that is not as strong as mine, or that is missing an important detail.
- Score 4 for each correct answer with an explanation that barely works, or works by chance, but is missing several essential details.
- Score 2 for each correct answer with no explanation.
- Score 0 for each answer with an explanation that does not work.
- Score 0 for each answer that differs from mine, even if it has a strong explanation. In this case, however, please make sure I get your explanation so that I can revise the test and your score. A score of 0 in this case is required to maintain the objectivity of the test since it is self-administered.

Points to remember

- Figures are drawn to scale as accurately as possible.
- All lines should be considered ideal lines (i.e., with no thickness) and they are all either perfectly straight or perfect circles.
- Size, relative proportions, and relative positions matter.
- There is (at least) one simple and correct explanation for each problem.
- Do not settle for an explanation that almost works. Find one that does work.
- The problems are not easy. It is probably a good idea to spend at least one hour on each before giving up.

Procedure to find out how many correct choices you have made

- Look up in table 1 the four numbers corresponding to your choices for the four problems. Cover table 2 while using table 1.
- Add the numbers up and make sure the sum is correct. Then, cover table 1 and uncover table 2.
- If the sum is in table 2, the left column will indicate how many correct choices you have made. If the sum is not in the table, you have made no correct choices. Now cover both tables and do not use them again.

Table 1

	A	B	C	D	E	F	G	H
Problem 1	17	99	81	113	145	65	33	129
Problem 2	162	146	100	50	194	34	98	146
Problem 3	115	35	163	99	131	179	51	117
Problem 4	116	132	164	100	84	22	52	68

Table 2

4 correct choices	116 120 134 136 164 166 198 208 216 224 226 276 328 338 340 354 358 376 424 450 466 484 528 536 544 552
3 correct choices	114 182 256 272 288 296 304 320 336 352 368 370 374 384 400 406 416 432 448 456 480 504 512 534 576 594
2 correct choices	174 190 206 222 238 254 270 286 302 318 334 350 366 382 398 414 430 434 446 462 478 494 510 526 542 574
1 correct choice	104 108 110 124 140 156 158 162 172 188 194 200 204 220 236 242 252 268 284 290 294 300 316 332 348 364 380 396 412 418 428 444 454 460 468 476 486 492 508 524 528 530 540 556 558 572 588 590 598 604 620 636

Do not read beyond this point until you are done working on the problems

Solutions

The solutions are provided as verbal explanations with no drawings and in small type to make it less likely that you would read them by accident. For the same reason, the letters for the correct solutions are provided in a roundabout way. Do not read them until you have written down your final choices and explanations. This is your last chance. Do not rush to read the explanations if you feel that you could improve your answers. You may regret it.

Problem no. 1: Consider figure I to be composed of two opaque parts (a vertical cross and a diagonal cross) one behind the other. Rotate the opaque part in the background 45 degrees (in either direction) around its center. Consider figure III to be composed of two opaque parts (a small square and a big square) one behind the other. Apply the same transformation to obtain the figure identified with the first letter in the word but. Option C can not be obtained by a similar transformation since the smaller square is in the foreground and it increases in size.

Problem no. 2: Consider figure I to be composed of two transparent parts (a square and a rectangle). Rotate one of the two parts 45 degrees (in either direction) around its center and shrink it by approximately 1.4142 (the square root of 2) preserving the location of its center. Consider figure III to be composed of two transparent parts (a diagonal cross and a square). Apply the same transformation to obtain the figure identified with the third letter in the word tuck. Option A can not be obtained by a similar transformation since the cross shrinks by approximately 2.8284 (twice the square root of 2).

Problem no. 3: Consider figure I to be composed of two opaque parts (a rectangle and a square, half of which is hidden by the rectangle). Slide the part in the background horizontally by exactly its horizontal length. Consider figure III to be composed of two opaque parts (an L shaped figure and a square, half of which is hidden by the L). Apply the same transformation to obtain the figure identified with the second letter in the word show.

Problem no. 4: Consider figure I to be composed of two transparent parts (two circles). Bisect both parts horizontally, enlarge the bottom part of the left part and the top part of the right part to make them twice as big (in length, not area). Expand them so that they stay on the same horizontal line, centered horizontally around their previous (not expanded) positions. Slide the two parts so transformed horizontally into each other by half the horizontal length of the original parts. Consider figure III to be composed of two triangles. Apply the same transformation to obtain the figure identified with the first letter of the word for. Options B and C can not be obtained by applying the most obvious transformation (i.e., one of the two circles doubles in size and moves) since the triangle in the middle shrinks.

Preliminary EIT Report

November 1993

This is a *preliminary* "statistical" report on the Eight Item Test (EIT), a test I developed. This test, along with its predecessor the Four Item Test (FIT), was published in *Noesis* (the journal of the Mega Society). Both tests were also mailed directly to those listed in the *Combined Membership Roster for the Four Sigma, Prometheus, Noetic, and Mega Societies*, a list with about 600 names published in the *Four Sigma Bulletin*, Number 2. The FIT was mailed to about 98% of those on the list and the EIT to about 60%. Both tests have also been distributed by other means and I estimate that by November '93 they have reached over a thousand highly intelligent people.

In all, I received a small number of responses to date: 10 for the EIT, and a little over 30 for the FIT. (I obtained a few additional preliminary responses for the EIT, but I excluded them from the analysis since they were obtained under questionable conditions. For example, from friends who took the FIT with whom I discussed some aspects of the test.) I clearly realize that a sample of 10 is almost laughable. However, there are several reasons why I decided to publish this report anyway.

The EIT was designed to be extremely difficult. In addition, the EIT is probably the most "self-selecting" test designed to study high intelligence. Since a fit for three figures has to be found for each problem, the testee receives a clear indication about how well he or she is doing. I expect that many people attempted the test but did not mail it in, because it became obvious to them that their score would be close to 0. Therefore, I hope that these 10 responses are just the tip of an iceberg of dozens of people who have attempted the test. I expect this effect to work "on top of" the usual lack of enthusiasm that people with *relatively* low ability (for this type of problem) exhibit towards completing tests such as the EIT. I expect that few people below the 99.999th percentile (1 in 100,000) would be interested in completing this test. In this light, 10 is not such a horrible number for a preliminary report. To put this number in perspective, notice that LAIT scores are still reported based on a sample of 553 testees. Among those, only 17 scored 165 or higher (that is, above the 99.9975 percentile)

I hope that the availability of these data will encourage more people to attempt the EIT.

Obviously, the limited number of responses rules out almost any standard statistical procedure on the data. Therefore I have not attempted to calculate correlations, to calculate detailed item-response curves, or to provide a norming for the test. Instead, most of the relevant information is summarized in the two tables below. Table 1 provides the scores of the 10 men (yes, all male) on the EIT, and when available, on the LAIT and Mega tests. (EIT scores are reported on a scale of 0 to 100.) In addition, the age and the time spent on the test are also reported. The time is broken down into up to two components separated by slashes: the first number is the time spent on the first attempt, the second number is the time spent on all attempts. (Multiple attempts are allowed.) Only two people made more than one attempt. In one case the score did not change significantly: 77 to 75; in the other case the score did change significantly: 38 to 67. In the second case, little time was spent on the first attempt.

Table 2 provides a very rough item-response analysis. For each item, the average score *for that item* (expressed as a percentage of the full score for that item) is provided in two groups: one without partial credits and one with partial credits. (Partial credits are given in the EIT. The average score without partial credits is equivalent to the percentage of respondents in that group that obtained a full credit answer.) Three numbers are provided for every group, one for each quarter of the EIT range, except for the range 25-49 (no scores in this range have been obtained). Finally, for every item, the number of responses currently on the scoring list that produce a non-zero score is indicated. Notice that only one item has an alternative response with full credit. Responses with partial credits produce a score of 1/2, 1/4, or 1/8 of the full score. (Not all valid answers are present in the 10 responses.)

Table 2 shows that the items seem to behave well. Keep in mind that the number of responses is too low for the numbers in this table to be reliable (in the statistical sense). The items in this table are sorted first by the average score (with partial credits) of the 75-100 group and by the average score of the 50-74 group. The items have been re-labeled "item a" through "item h," based on this order. Thus, item a is probably the most difficult and item h is probably the easiest one.

A few additional comments: (a) There is a good match between LAIT scores and EIT scores. (b) Eight of the respondents were on the *Combined Membership Roster* described above, and two of them are Mega members. (c) No responses from members of Triple Nine, Mensa, or any other high-IQ societies below the four sigma level were received, even though several of them requested copies of the EIT.

Preliminary EIT Report

November 1993

Table 1

EIT ¹	LAIT ²	Mega ³	Age	Time ⁴
91%	168		33	20
77%	170		60	22/40
67%	168		34	7/25
63%			50	20
58%			27	20
50%	168		27	
22%	161	37	62	
16%	160	43	49	5
16%			60?	20+
0%	160	36	52	8

Table 2⁵

	Without Partial Credits ⁶			With Partial Credits ⁷			No. of Valid Answers ⁸	
	0-24%	50-74%	75-100%	0-24%	50-74%	75-100%	Full Credit ⁹	Partial Credit ¹⁰
Item a ¹¹	0%	0%	0%	0%	9%	13%	1	3
Item b	0%	0%	50%	3%	6%	56%	1	2
Item c	25%	0%	100%	38%	38%	100%	1	2
Item d	0%	50%	100%	0%	53%	100%	1	1
Item e	0%	75%	100%	12%	75%	100%	1	1
Item f	0%	75%	100%	0%	81%	100%	2	1
Item g	0%	100%	100%	0%	100%	100%	1	1
Item h	50%	100%	100%	53%	100%	100%	1	1

1. Percentage (100% represents a perfect score).
2. IQ score (mean 100, standard deviation 16) based on Second Norming.
3. Number of correct responses out of a total of 48 problems.
4. In hours; the first number represents time of first attempt, the second number represents the time spent on all attempts.
5. No scores were obtained in the 25-49% range. Therefore, only three of the four ranges of scores appear in the table.
6. Numbers in these three columns are equivalent to the percentage of respondents in each category obtaining a full credit answer.
7. Numbers in these three columns are slightly higher than the percentage of respondents in each category obtaining a full credit answer, since partial credits are also included. For the purpose of computing this number, one respondent with a 1/2, 1/4, or 1/8 credit answer is treated as 1/2, 1/4, or 1/8 of a respondent with a full score answer.
8. These numbers indicate the number of answers currently on the scoring list for which full or partial credit is given. The scoring list is updated as additional explanations are received.
9. All full credit answers contribute equally towards the total score. Each of them provides 12.5 points (on a scale of 0 to 100).
10. Each partial credit answer contributes 1/2, 1/4, or 1/8 of a full credit answer, depending on its quality.
11. The items have been re-labeled in order of difficulty. Item a seems to be the most difficult and item h seems to be the easiest.

About the EIT, FIT, and SSFIT

I designed the EIT (Eight Item Test) to study the ability to postulate theories to account for observed facts and to select the "best" ones among them. I chose to use figure analogies for this purpose because they seem to require little previous knowledge and seem easily understandable by most people.

I designed the problems so as to discriminate at very high levels of ability. The ideal way to ensure high discrimination, of course, is to test very large populations with known ability distributions. Unfortunately, this approach is rarely practical. A more questionable, but much easier, alternative is to test populations of individuals who have demonstrated very high ability on similar problem-solving tasks. I chose the latter approach. I have distributed the test to a large portion of the individuals who scored at or above an IQ of 164 on either the LAIT (Langdon Adult Intelligence Test, by Kevin Langdon) or the Mega Test (by Ronald Hoeflin). These are the best-known high-level IQ tests and are currently used for admission by IQ societies with very high IQ requirements (at or above IQ 164). I have also circulated the test among several other populations. Preliminary (and scant) data seem to confirm that the EIT discriminates from somewhere above 1 in 1,000 (corresponding to an IQ of 149) to somewhere above 1 in 1,000,000 (corresponding to an IQ of 176). I am very interested in collecting more data and I hope to be able to provide a tentative norming soon.

The relationship among the EIT, FIT, and SSFIT is as follows: the EIT is the primary test, the one I am interested in collecting data for. The FIT is a precursor and a subset of the EIT. I am not scoring it any more (with a few exceptions); I encourage tessees to try the EIT instead. The SSFIT (Self-Scoring Four Item Test) is a much easier test and is provided mainly as a training tool for the EIT. I created it to help compensate for differences in training and amount of previous exposure to figure analogy tests.

All items on the EIT have the same value towards the total score. Partial credit is given for non-perfect answers. All tessees receive the same credit for a given three-letter answer regardless of whether an explanation is included and regardless of its quality (see exception below). Good explanations are used to maintain a list of valid answers and their corresponding scores. (More than one answer is valid for each item on the EIT, although not all of them produce full credit.) However, if an answer is proved to be valid, all previous scores are not immediately adjusted. As I receive more data, I will freeze scoring sets for relatively long periods of time, and new normings may be produced as required. (The list of valid answers is fairly stable; few alternative solutions have been found so far and most of them do not produce full credit.)

To the best of my knowledge, I introduced a novel approach for multiple-choice tests with the the EIT. By requiring three-letter answers, several benefits are realized:

- (a) The probability of a correct guess is drastically minimized, without loss of the important benefits of a multiple-choice format ("objectivity," ease in scoring, clear indication to the tessees of the range of valid answers, etc.)
- (b) Improved feedback is provided to the tessees about the "correctness" of his or her choices.
- (c) Alternative "valid" answers become more rare.
- (d) Alternative "valid" answers of varying qualities (especially those not originally recognized by the test author) become easier to handle.
- (e) Higher discrimination is achieved for a given number of answers, due to (a).

Points (c) and (d) are particularly important. It is extremely difficult to "weed-out" all reasonable alternative answers, especially in tests directed to individuals with a very high level of ability. Additional valid explanations are often discovered by the tessees. The EIT provides an elegant mechanism to handle this. Even though the EIT has few items, it seems very unlikely that somebody would obtain a score above 25% by chance rather than high ability. Even with, for example, two full-credit answers per problem, four half-credit answers, eight quarter-credit answers, and so forth, for each problem, fewer than 1 in 30,000 of all possible answer patterns would have a score of 25% or more, fewer than 1 in 10 million would have a score of 38% or more, and fewer than 1 in 10 billion would have a score of 50% or more.

I chose eight items as a reasonable compromise between precision and required effort. At first sight it seems that a test with a very high number of items would be more accurate in identifying high general ability. However, few people are willing to spend much time taking tests. Therefore a high score depends not only on a high general ability but also on a high willingness to spend time on tests. It seems likely that the LAIT (56 items) and especially the Mega Test (48 items) exhibit this effect to some extent. These tests, although usually considered "untimed," are still, in a way, timed tests. Most people end up spending between 10 and 100 hours on them and those who spend the most time increase their likelihood of a high score. This seems difficult to avoid, and maybe there is no such thing as a truly "untimed" test (a test where huge investments of time beyond a certain maximum would have little effect on the final score). If the LAIT and Mega have a "maximum" time beyond which little additional gain would be made in total score, I would expect it to be about 50 hours for the LAIT and 200 hours for the Mega Test (this is a very rough wild guess). I hope (and I have some indication) that such a "maximum" time for the the EIT is about 20 hours.

Both the FIT and the EIT have been published in Noesis, the journal of the Mega Society. The highest scores on the EIT so far are 91%, 77%, 67%, and 63% (the last one by a member of the Mega society, a high-IQ society that used to claim discrimination at the one in a million level). If there were to be a perfect correlation between EIT scores and IQ, I expect that a rough (and probably conservative) equivalence would be $IQ = 140 + EIT/2$ where EIT represents the score on the EIT on a scale of 0 to 100. IQs below about 150 would be considered "below the valid range of the test."

Any efforts to help me distribute the test will be appreciated (remember, however, that the copyright notice *must* be included and the test should not be reduced). Also, please help me by not distributing or discussing answers to the problems. And, of course, try it. You can obtain these tests by sending me (Aax) a SASE and \$2 at Box 1391, Princeton, NJ 08542.

Figure Analogies and Intelligence

An important function of intelligence is to make useful predictions. Predictions can be made by creating theories to fit the observed facts. When a theory fits all the observed facts it can be deemed likely to also fit new facts. Thus it can be used to make *predictions*.

For example, Archimedes's theory of submerged bodies states that the lift experienced by a submerged body is equal to the weight of the displaced liquid. This theory fits the known fact that all bodies with an overall density higher than the density of water will sink in water if no other forces act on them. If we are given an object that we have never seen before, and we are accurately informed that its density is higher than water's density, we can accurately *predict* that it will sink in water.

We usually think of theories in connection with science. However, theories (of varied levels of sophistication) are constructed by all beings that exhibit some intelligence. In particular, humans are constantly (consciously and unconsciously) making theories to explain the behavior of their surroundings. Therefore, one way to test intelligence is to test the ability to create useful theories. Analogy problems are particularly useful in this regard.

Analogy problems usually require you to find a relationship among pairs of objects. Usually two pairs are used. The first pair is presented complete, but only the first object of the second pair is presented. The goal is to determine what object would "fit" as the second object of the second pair. To solve this problem, the testee has to construct a theory that would allow him or her to "predict," given the first object of the pair, what the second object would be. For example, given the problem: "dog" is to "leg" as "car" is to ?, "wheel" would be a possible correct answer. (The "theory" in this case could be "One of the several similar objects on which the object stands or moves in normal operation.") In multiple-choice problems, a set of possible answers is presented and the theory should "predict" one and only one of them.

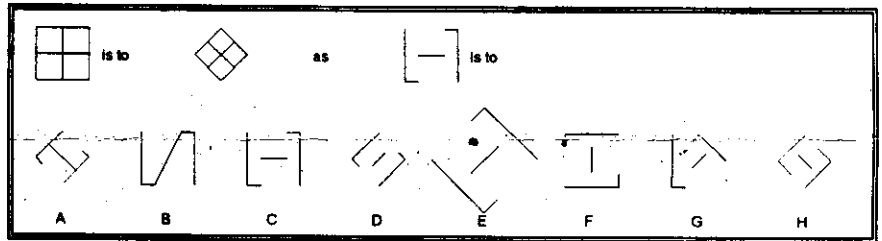
An infinite number of theories can be created to accommodate the facts and obtain an answer. In a multiple-choice problem it is *always* possible to construct a theory for every possible answer. For example, "kor" could be provided as the answer to the previous example. The supporting "theory" would be "replace the first letter ('d') with a letter 8 spaces down the alphabet ('l'), the second letter ('o') with a vowel three spaces down ('e') (including wrap-around, that is going back to 'a' after the 'u'), and leave the third letter unchanged."

The "best" theory should be selected. As in real life or science, "best" is usually the simplest and most general theory which explains *all* of the observed facts. Simple and general theories usually apply to more objects and therefore have a higher predictive value. "Best" is not easy to define and I will not attempt to do so. Finally, great care has to be taken to discard those theories that do not fit the known facts, even when they "almost" do.

Figure analogies are a good class of items because they require little specialized knowledge. They require knowledge of only a few basic operations that most people understand (even among widely different cultures). To solve a two-pair figure analogy problem, try

to construct theories that allow you to construct the second object of the first pair, given the first. If it is a multiple-choice problem, discard all the theories that do not point to one and only one of the answers. Among the remaining theories, choose the "best." Make sure your theory explains *all* of the observed facts.

On the right there is a sample figure analogy problem. I strongly recommend that you look at it and try to solve it before reading any further.



For this problem, we will only analyze theories where parts of each object rotate and parts of each object (possibly the same parts) change size. To simplify the discussion, I will number the three objects on the first row 1, 2, and 3, from left to right. That is, the first pair consists of objects 1 and 2, and the second pair consists of 3 and one of A through H.

The simplest theory, that the object rotates 45 degrees and shrinks by about 1.4142 (the square root of 2) seems to point to (H). In (H), however, the short lines at the end of the long lines are no longer a fourth of the length of the long lines. Since our theory does not account for this fact, it has to be modified or discarded. (E) cannot be explained by this theory, either, since (3) cannot be transformed by the rules of this theory into (E). The fact that (E) can be transformed into (3) is irrelevant, since the *direction* of the relationship matters.

We can explain (C) and (F) by postulating that the angle of rotation and the change in size are not the same between the objects of the first pair and the objects of the second pair. However, this gives us a very weak theory with almost no predictive value, since we have no way to determine what angle of rotation and change of size to apply to a given first object of a pair. Similarly, (G) can be explained by postulating that, sometimes, only part of the object rotates and shrinks.

(A) can be explained by postulating that the external part of the object rotates and shrinks and the center rotates and grows but is "clipped" to the boundaries of the external part, as if the external part were a window through which we see the internal part. However, the external part of (3) does not look quite like a window and, in general, arbitrary postulation of clipping of lines should be avoided. Nevertheless, this is so far our strongest theory. Since it is not very good, we should look for a better one. If we do not find one, (A) should be our answer.

Fortunately, we can explain (D) by postulating that the external part of the first object rotates 45 degrees clockwise, while the internal part rotates 45 degrees counterclockwise, and both shrink by about 1.4142 (the square root of 2). We find no reasonable theory to account for (B), and therefore our answer is (D).

Sometimes you might be able to find two or more good theories pointing to two different answers. Sometimes you might not be able to find any good theory. This might be due to an error or inability to understand the problem on your part, or it might be due to an error in the test. Even when the author of the test carefully examines the problems, errors might remain. Also, alternative solutions, not foreseen by the author, might be found. A good test should be carefully "cleaned up" by obtaining feedback from many testees.

I have developed two tests based exclusively on figure analogies: the SSFIT and the EIT. The SSFIT is self-scoring and moderately difficult. The EIT is not self-scoring and is *very* difficult. You can obtain either test by sending me a SASE and \$2 (Aax, Box 1391, Princeton NJ 08542).